

ANALYSIS OF GABLE FRAMES,  
BY  
ELECTRONIC COMPUTER

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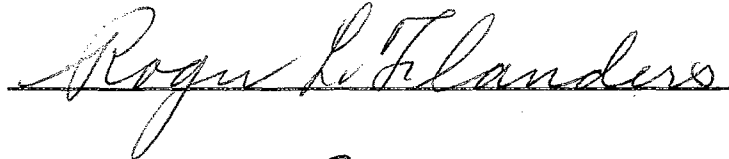
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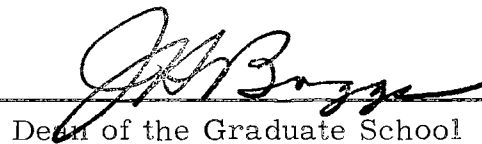
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## NOMENCLATURE

$H_{ij}$ . . . . .	Horizontal thrust or shear at i on member ij
$I$ . . . . .	Moment of inertia
$L$ . . . . .	Length of span
$M_{ij}$ . . . . .	Moment at i on member ij
$P_i$ . . . . .	Horizontal force at joint i
$V_{ij}$ . . . . .	Vertical thrust or shear at i on member ij
$w$ . . . . .	Intensity of distributed load
$\alpha$ . . . . .	Column length parameter
$\beta$ . . . . .	Gable height parameter
$r_2, r_4, r_6, r_8$ . . . . .	Column moment of inertia parameters
$r_1, r_3, r_5, r_7$ . . . . .	Gable moment of inertia parameters
$v$ . . . . .	$\sqrt{1 + 4\beta^2}$

## CHAPTER I

### INTRODUCTION

#### 1-1. General

It is recognized that the analysis of continuous frames is a time-consuming and laborous process. Fortunately, the use of electronic digital computers can save considerable engineering manpower in performing the analysis and design computations. In practical design, the designer is able to assume a trial design and immediately check the sufficiency of the design at will, providing he has access to an applicable computer program.

The purpose of this thesis is to develop a general computer program for use in analyzing continuous gable frames. The structures considered are one-, two-, three-, and four-span gable frames with different moments of inertia for different members. The bottoms of the frames are assumed to be hinged and all dimensions are expressed as functions of the equal span lengths (Fig. 1-1). The frames are assumed to be acted on by

- (1) Uniformly distributed load
- (2) Horizontal force applied at each joint independently.

The method used for analyzing these frames is the string polygon method. Assumptions common to general structural analysis apply. The sign convention of the three-moment equation is adopted. The end bending moments and end slopes are positive if they cause tension on the dotted side of the member. The elastic weights are



positive if acting in the positive direction of the z-axis.

Typical elasto-static and static equations are presented in matrix form, in terms of parameters  $\alpha$ ,  $\beta$ ,  $\nu = \sqrt{1+4\beta^2}$ , and r's. Combinations of these parameters are suggested which might be expected to cover a practical range, if moment coefficient tables are desired as a future extension of this thesis. End moments due to unit loadings are yielded directly by solving the matrix equations presented.

## 1-2. Background

Moment coefficients for gable frames were first developed and presented in tabular form by Gillespie (1, 2) and Gillespie and Tuma(3). Similar coefficients for continuous frames with curved girders were presented by Carmen (4) and Larkin (5). However, these developments and presentations were limited to frames with constant moment of inertia for all members.

Moment coefficients for one- and two-span symmetrical frames with different moments of inertia for different members have been developed by Hale (6). Coefficient charts for the analysis of single span rectangular, gable, and parabolic frames were developed by Griffiths (7). Moment tables for one-span symmetrical gable frames were presented by Korn (8).

The string polygon method of analysis, adopted for use in this development, is based on a generalization of the joint elastic weight expression. A general approach was presented by Jan J. Tuma in his lectures at Oklahoma State University and extended by Chu (10), Oden (11), Boecker (12), Yu (13), and others (14, 15) to the solution of many special problems. The application of the string polygon

method of analysis to complex frames has been presented by Tuma and Oden (9).

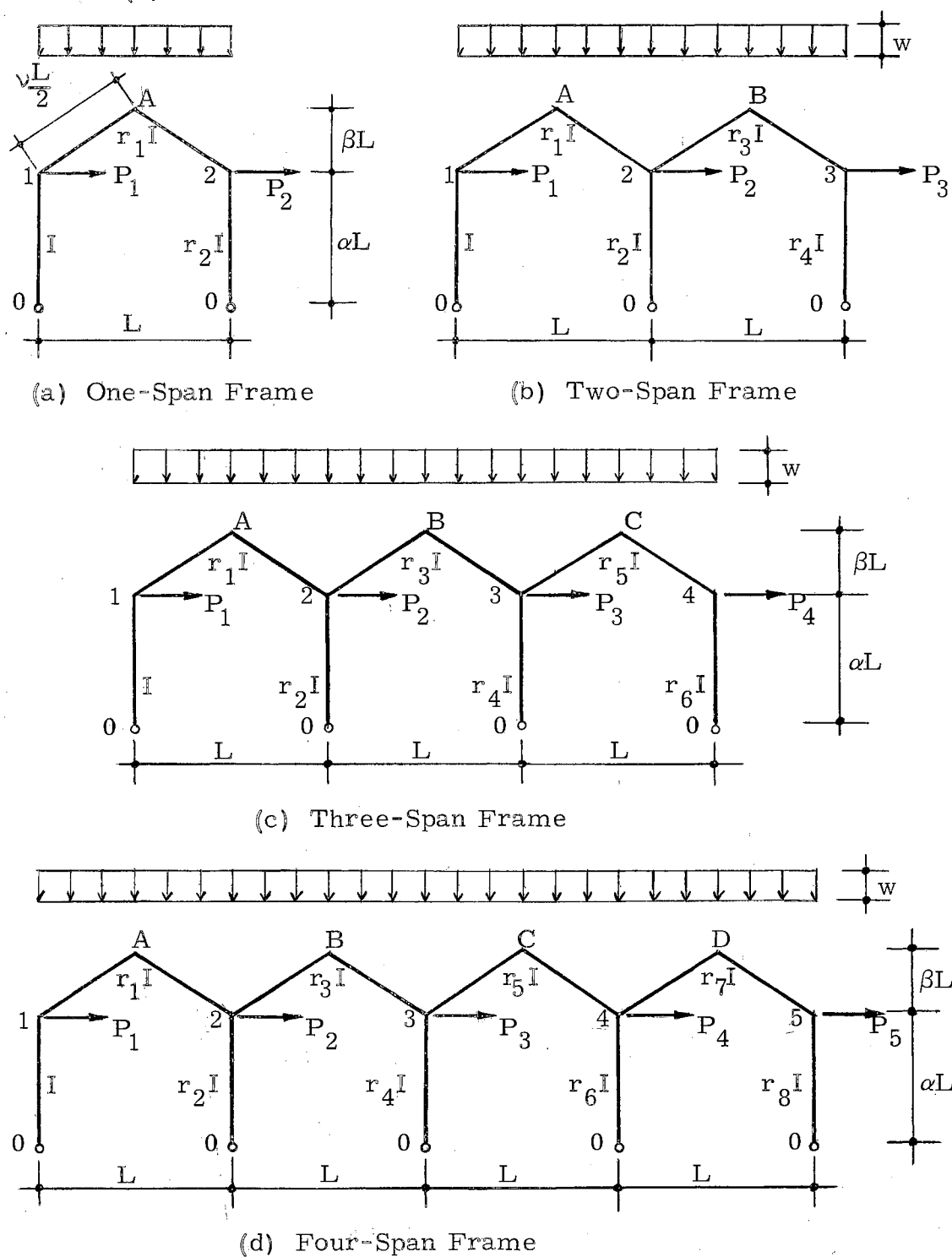


FIG. 1-1. TYPICAL GABLE FRAMES

## CHAPTER II

### THEORY

#### 2-1. Deformation

A four-span gable frame with base hinged is considered (Fig. 2-1). Moments at the ends of all members are selected as unknowns (Fig. 2-2). Directions of these bending moments are selected in such a way that compatibility between adjacent panels is satisfied. Moments causing tension on the dotted side of the member are positive.

Deformation equations for two adjacent beams  $ij$  and  $jk$  acted on by a general system of loads are given as follows:

$$\begin{aligned}\bar{P}_{ji} &= \phi_{ji} = M_j F_{ji} + M_i G_{ij} + \tau_{ji} \\ \bar{P}_{jk} &= \phi_{jk} = M_j F_{jk} + M_k G_{kj} + \tau_{jk}\end{aligned}$$

from which

$$\bar{P}_j = \phi_j = \phi_{ji} + \phi_{jk} = M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \Sigma \tau_j$$

where

$$\begin{aligned}\bar{P}_{ji}, \bar{P}_{jk} &= \text{Segmental elastic weights at } j \text{ of beams } ij \text{ and } jk, \\ &\quad \text{respectively.} \\ \bar{P}_j &= \text{Joint elastic weight at } j. \\ \phi_{ji}, \phi_{jk} &= \text{Slope at } j \text{ of beams } ij \text{ and } jk, \text{ respectively.} \\ \phi_j &= \text{Change in slope at } j \text{ of two adjacent beams } ij \text{ and } jk. \\ M_i, M_j, M_k &= \text{Moments at } i, j, \text{ and } k, \text{ respectively.} \\ F_{ji}, F_{jk} &= \text{Angular flexibilities} \\ G_{ij}, G_{kj} &= \text{Angular carry-over values}\end{aligned}$$

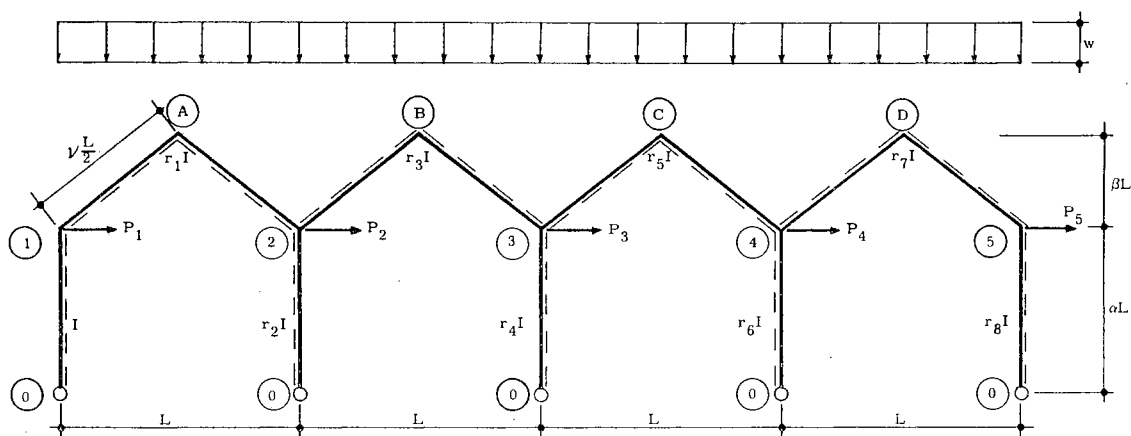


FIG. 2-1. FOUR-SPAN FRAME

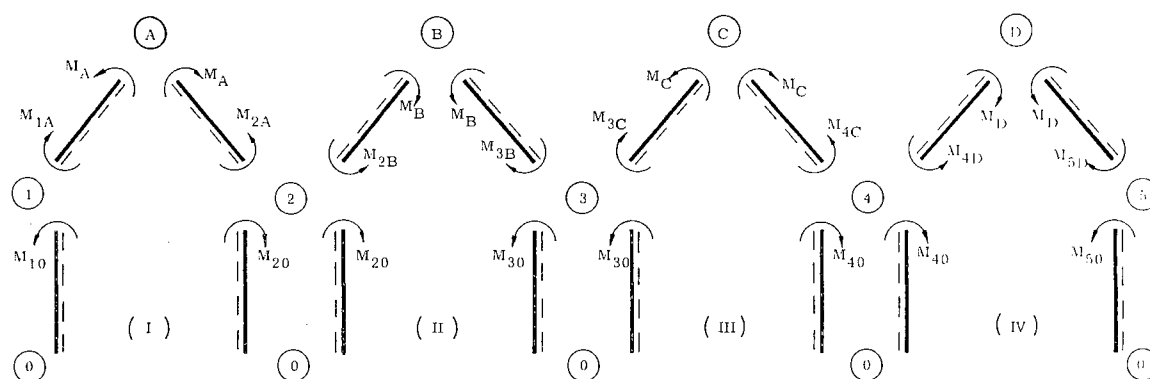


FIG. 2-2. REAL PANELS

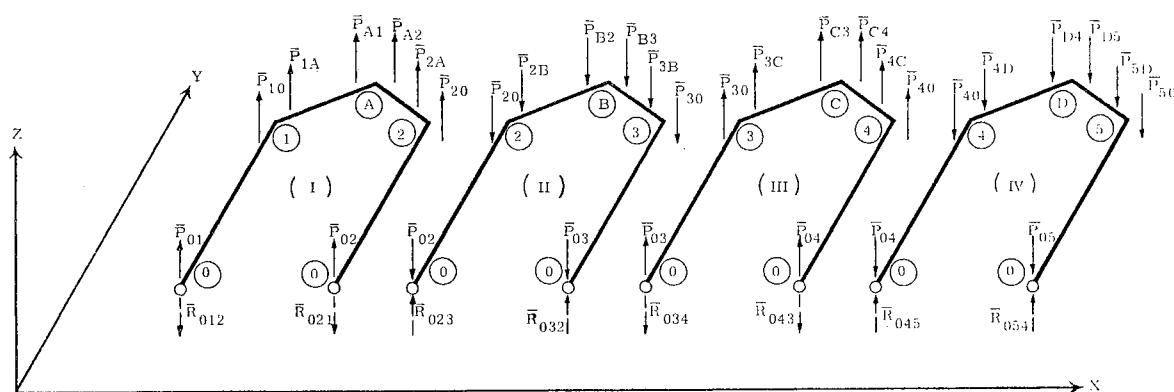


FIG. 2-3. CONJUGATE PANELS

TABLE 2-1. (See Ref. 14, 11) ANGULAR FUNCTIONS

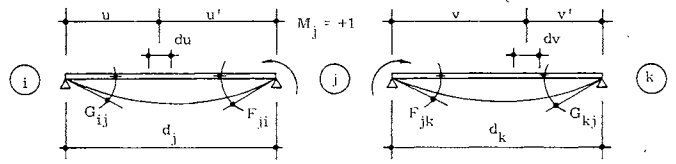
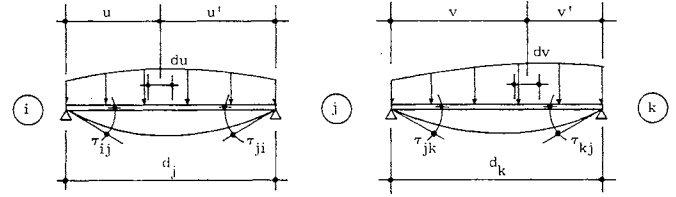
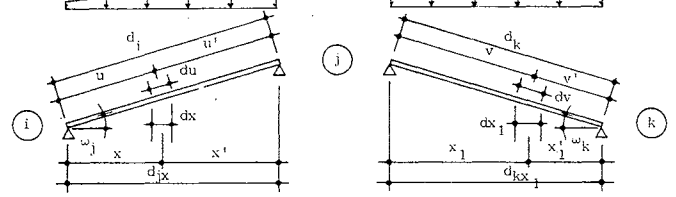
	Quantity	Algebraic Definition	Physical Interpretation
 <p>Angular Flexibilities and Carry-Over Values</p>	$F_{ji}$          $G_{ij}$          $G_{kj}$	$\int_i^j \frac{u^2 du}{d_j^2 EI_u}$   $\int_j^k \frac{v^2 dv}{d_k^2 EI_v}$   $\int_i^j \frac{uu' du}{d_j^2 EI_u}$   $\int_k^j \frac{vv' dv}{d_k^2 EI_v}$	<p>End slope at j of the simple beam ij due to unit moment applied at j.</p>          <p>End slope at j of the simple beam jk due to unit moment applied at j.</p>          <p>End slope at i of the simple beam ij due to unit moment applied at j.</p>          <p>End slope at k of the simple beam kj due to unit moment applied at j.</p>
 <p>Angular Load Function</p>	$\tau_{ji}$          $\tau_{jk}$	$\int_i^j \frac{BM_u u du}{d_j EI_u}$   $\int_j^k \frac{BM_v v' dv}{d_k EI_v}$	<p>End slope at j of the simple beam ij due to loads.</p>          <p>End slope at j of the simple jk due to loads.</p>
 <p>Angular Load Function for Vertical Loading</p>	$\tau_{ji}$          $\tau_{jk}$	$\frac{1}{\cos \omega_j} \int_i^j \frac{BM_u x dx}{d_{jx} EI_u}$   $\frac{1}{\cos \omega_k} \int_j^k \frac{BM_v x_1 dx_1}{d_{kx_1} EI_v}$	<p>End slope at j of the simple beam ij due to vertical loads.</p>          <p>End slope at j of the simple beam jk due to vertical loads.</p>

TABLE 2-2. ANGULAR FUNCTIONS			
MEMBER	F	G	$\tau$
$\textcircled{0} \textcircled{1} = \textcircled{1} \textcircled{0}$	$\frac{\alpha L}{3E(I)}$	$\frac{\alpha L}{6E(I)}$	
$\textcircled{1} \textcircled{A} = \textcircled{A} \textcircled{1}$	$\frac{\nu \frac{L}{2}}{3E(r_1 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_1 I)}$	$+\frac{w\nu(\frac{L}{2})^3}{24E(r_1 I)}$
$\textcircled{A} \textcircled{2} = \textcircled{2} \textcircled{A}$	$\frac{\nu \frac{L}{2}}{3E(r_1 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_1 I)}$	$+\frac{w\nu(\frac{L}{2})^3}{24E(r_1 I)}$
$\textcircled{2} \textcircled{0} = \textcircled{0} \textcircled{2}$	$\frac{\alpha L}{3E(r_2 I)}$	$\frac{\alpha L}{6E(r_2 I)}$	
$\textcircled{2} \textcircled{B} = \textcircled{B} \textcircled{2}$	$\frac{\nu \frac{L}{2}}{3E(r_3 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_3 I)}$	$-\frac{w\nu(\frac{L}{2})^3}{24E(r_3 I)}$
$\textcircled{B} \textcircled{3} = \textcircled{3} \textcircled{B}$	$\frac{\nu \frac{L}{2}}{3E(r_3 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_3 I)}$	$-\frac{w\nu(\frac{L}{2})^3}{24E(r_3 I)}$
$\textcircled{3} \textcircled{0} = \textcircled{0} \textcircled{3}$	$\frac{\alpha L}{3E(r_4 I)}$	$\frac{\alpha L}{6E(r_4 I)}$	
$\textcircled{3} \textcircled{C} = \textcircled{C} \textcircled{3}$	$\frac{\nu \frac{L}{2}}{3E(r_5 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_5 I)}$	$+\frac{w\nu(\frac{L}{2})^3}{24E(r_5 I)}$
$\textcircled{C} \textcircled{4} = \textcircled{4} \textcircled{C}$	$\frac{\nu \frac{L}{2}}{3E(r_5 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_5 I)}$	$+\frac{w\nu(\frac{L}{2})^3}{24E(r_5 I)}$
$\textcircled{4} \textcircled{0} = \textcircled{0} \textcircled{4}$	$\frac{\alpha L}{3E(r_6 I)}$	$\frac{\alpha L}{6E(r_6 I)}$	
$\textcircled{4} \textcircled{D} = \textcircled{D} \textcircled{4}$	$\frac{\nu \frac{L}{2}}{3E(r_7 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_7 I)}$	$-\frac{w\nu(\frac{L}{2})^3}{24E(r_7 I)}$
$\textcircled{D} \textcircled{5} = \textcircled{5} \textcircled{D}$	$\frac{\nu \frac{L}{2}}{3E(r_7 I)}$	$\frac{\nu \frac{L}{2}}{6E(r_7 I)}$	$-\frac{w\nu(\frac{L}{2})^3}{24E(r_7 I)}$
$\textcircled{5} \textcircled{0} = \textcircled{0} \textcircled{5}$	$\frac{\alpha L}{3E(r_8 I)}$	$\frac{\alpha L}{6E(r_8 I)}$	

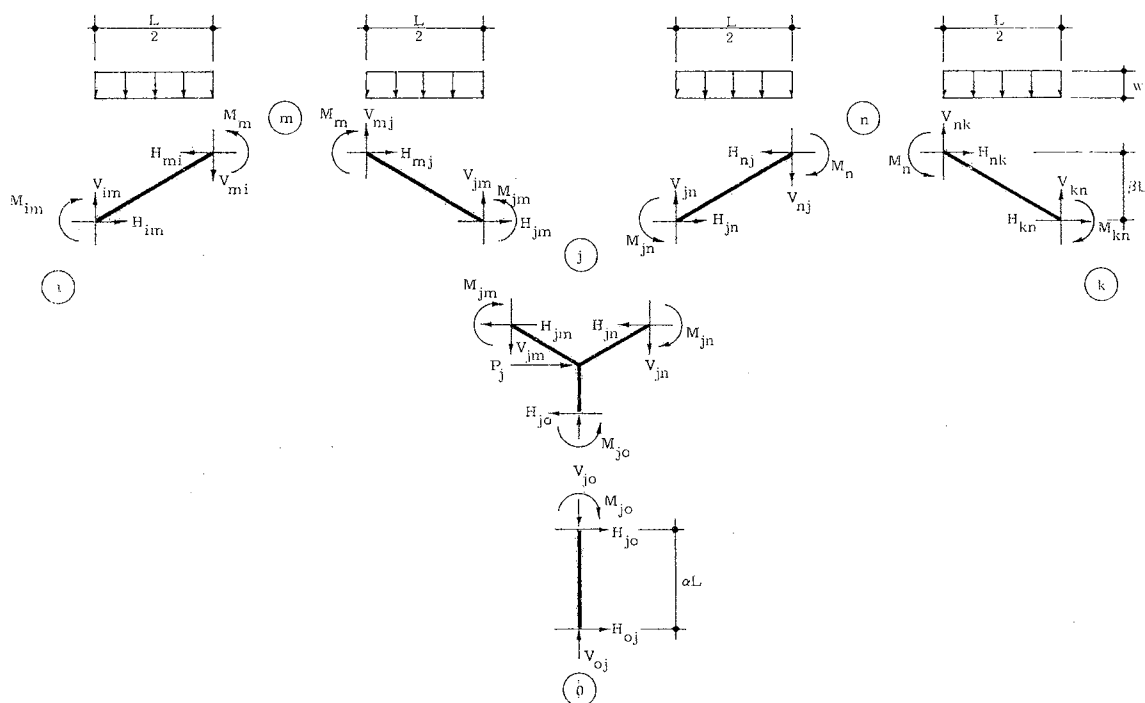


FIG. 2-4. A TYPICAL JOINT

TABLE 2-3. ELASTIC WEIGHTS

Member	Elastic Wt.	M <sub>10</sub>	M <sub>1A</sub>	M <sub>A</sub>	M <sub>2A</sub>	M <sub>20</sub>	M <sub>2B</sub>	M <sub>B</sub>	M <sub>3B</sub>	M <sub>30</sub>	M <sub>3C</sub>	M <sub>C</sub>	M <sub>4C</sub>	M <sub>40</sub>	M <sub>4D</sub>	M <sub>D</sub>	M <sub>5D</sub>	M <sub>50</sub>	W
(0) (1)	$\bar{P}_{01}$	$\frac{\alpha L}{6EI}$																	
	$\bar{P}_{10}$	$\frac{\alpha L}{3EI}$																	
(1) (A)	$\bar{P}_{1A}$		$\frac{\sqrt{L/2}}{3E(r_1I)}$	$\frac{\sqrt{L/2}}{6E(r_1I)}$															$\frac{\sqrt{L/2}^3}{24E(r_1I)}$
	$\bar{P}_{A1}$		$\frac{\sqrt{L/2}}{6E(r_1I)}$	$\frac{\sqrt{L/2}}{3E(r_1I)}$															$\frac{\sqrt{L/2}^3}{24E(r_1I)}$
(A) (2)	$\bar{P}_{A2}$			$\frac{\sqrt{L/2}}{3E(r_1I)}$	$\frac{\sqrt{L/2}}{6E(r_1I)}$														$\frac{\sqrt{L/2}^3}{24E(r_1I)}$
	$\bar{P}_{2A}$			$\frac{\sqrt{L/2}}{6E(r_1I)}$	$\frac{\sqrt{L/2}}{3E(r_1I)}$														$\frac{\sqrt{L/2}^3}{24E(r_1I)}$
(2) (0)	$\bar{P}_{20}$					$\frac{\alpha L}{3E(r_2I)}$													
	$\bar{P}_{02}$					$\frac{\alpha L}{6E(r_2I)}$													
(2) (B)	$\bar{P}_{2B}$						$\frac{\sqrt{L/2}}{3E(r_3I)}$	$\frac{\sqrt{L/2}}{6E(r_3I)}$											$-\frac{\sqrt{L/2}^3}{24E(r_3I)}$
	$\bar{P}_{B2}$						$\frac{\sqrt{L/2}}{6E(r_3I)}$	$\frac{\sqrt{L/2}}{3E(r_3I)}$											$-\frac{\sqrt{L/2}^3}{24E(r_3I)}$
(B) (3)	$\bar{P}_{B3}$							$\frac{\sqrt{L/2}}{3E(r_3I)}$	$\frac{\sqrt{L/2}}{6E(r_3I)}$										$-\frac{\sqrt{L/2}^3}{24E(r_3I)}$
	$\bar{P}_{3B}$							$\frac{\sqrt{L/2}}{6E(r_3I)}$	$\frac{\sqrt{L/2}}{3E(r_3I)}$										$-\frac{\sqrt{L/2}^3}{24E(r_3I)}$
(3) (0)	$\bar{P}_{30}$									$\frac{\alpha L}{3E(r_4I)}$									
	$\bar{P}_{03}$									$\frac{\alpha L}{6E(r_4I)}$									
(3) (C)	$\bar{P}_{3C}$								$\frac{\sqrt{L/2}}{3E(r_5I)}$	$\frac{\sqrt{L/2}}{6E(r_5I)}$									$\frac{\sqrt{L/2}^3}{24E(r_5I)}$
	$\bar{P}_{C3}$								$\frac{\sqrt{L/2}}{6E(r_5I)}$	$\frac{\sqrt{L/2}}{3E(r_5I)}$									$\frac{\sqrt{L/2}^3}{24E(r_5I)}$
(C) (4)	$\bar{P}_{C4}$									$\frac{\sqrt{L/2}}{3E(r_5I)}$	$\frac{\sqrt{L/2}}{6E(r_5I)}$								$\frac{\sqrt{L/2}^3}{24E(r_5I)}$
	$\bar{P}_{4C}$									$\frac{\sqrt{L/2}}{6E(r_5I)}$	$\frac{\sqrt{L/2}}{3E(r_5I)}$								$\frac{\sqrt{L/2}^3}{24E(r_5I)}$
(4) (0)	$\bar{P}_{40}$													$\frac{\alpha L}{3E(r_6I)}$					
	$\bar{P}_{04}$													$\frac{\alpha L}{6E(r_6I)}$					
(4) (D)	$\bar{P}_{4D}$														$\frac{\sqrt{L/2}}{3E(r_7I)}$	$\frac{\sqrt{L/2}}{6E(r_7I)}$			$-\frac{\sqrt{L/2}^3}{24E(r_7I)}$
	$\bar{P}_{D4}$														$\frac{\sqrt{L/2}}{6E(r_7I)}$	$\frac{\sqrt{L/2}}{3E(r_7I)}$			$-\frac{\sqrt{L/2}^3}{24E(r_7I)}$
(D) (5)	$\bar{P}_{D5}$															$\frac{\sqrt{L/2}}{3E(r_7I)}$	$\frac{\sqrt{L/2}}{6E(r_7I)}$		$-\frac{\sqrt{L/2}^3}{24E(r_7I)}$
	$\bar{P}_{5D}$															$\frac{\sqrt{L/2}}{6E(r_7I)}$	$\frac{\sqrt{L/2}}{3E(r_7I)}$		$-\frac{\sqrt{L/2}^3}{24E(r_7I)}$
(5) (0)	$\bar{P}_{50}$																	$\frac{\alpha L}{3E(r_8I)}$	
	$\bar{P}_{05}$																	$\frac{\alpha L}{6E(r_8I)}$	

$$\tau_{ji}, \tau_{jk} = \text{Angular load functions.}$$

$$\Sigma F_j = F_{ji} + F_{jk}$$

$$\Sigma \tau_j = \tau_{ji} + \tau_{jk}$$

The angular flexibilities, carry-over values, and load functions are defined in Table 2-1. Table 2-1 is taken from Natarajan (14) and Oden (11) and is shown here for completeness. For all members of the four-span gable frame, these quantities are calculated and listed in Table 2-2.

## 2-2. Elasto-Static Equations

The joint elastic weight  $\bar{P}_j$  is defined as the change in slope of the adjacent beams  $ij$  and  $jk$  at  $j$ . The graphical representation of this change in slope is the force vector acting perpendicular to the plane of the beams. Consequently, they represent a new set of force vectors in a state of equilibrium. Elastic weights for all members of the four-span gable frame are calculated and shown in Table 2-3.

Because each panel is a closed string polygon, the frame may be resolved into the same number of conjugate panels as that of the real frame (Fig. 2-3). For any panel, elasto-static equations may be written in many different forms. In this study, four moment equilibrium equations and three reactive equilibrium equations are utilized.

There are 17-unknowns (end moments) in this gable frame; therefore, 17 equations are necessary for their solution. Ten equations based on the static equilibrium are available; thus, seven additional equations must be obtained from deformation compatibility.



The elasto-static equations are (Fig. 2-3).

$$\begin{aligned} \Sigma \bar{M}_{00(I)} &= 0 \\ \alpha L [\bar{P}_{10} + \bar{P}_{1A} + \bar{P}_{2A} + \bar{P}_{20}] + (\alpha + \beta) L [\bar{P}_{A1} + \bar{P}_{A2}] &= 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \Sigma \bar{M}_{00(II)} &= 0 \\ \alpha L [\bar{P}_{20} + \bar{P}_{2B} + \bar{P}_{3B} + \bar{P}_{30}] + (\alpha + \beta) L [\bar{P}_{B2} + \bar{P}_{B3}] &= 0 \end{aligned} \quad (2)$$

$$\begin{aligned} \Sigma \bar{M}_{00(III)} &= 0 \\ \alpha L [\bar{P}_{30} + \bar{P}_{3C} + \bar{P}_{4C} + \bar{P}_{40}] + (\alpha + \beta) L [\bar{P}_{C3} + \bar{P}_{C4}] &= 0 \end{aligned} \quad (3)$$

$$\begin{aligned} \Sigma \bar{M}_{00(IV)} &= 0 \\ \alpha L [\bar{P}_{40} + \bar{P}_{4D} + \bar{P}_{5D} + \bar{P}_{50}] + (\alpha + \beta) L [\bar{P}_{D4} + \bar{P}_{D5}] &= 0 \end{aligned} \quad (4)$$

$$\begin{aligned} \Sigma \bar{R}_{0(I, II)} &= 0 \\ \bar{R}_{021} - \bar{R}_{023} &= 0 \end{aligned} \quad (5)$$

$$\begin{aligned} \Sigma \bar{R}_{0(II, III)} &= 0 \\ \bar{R}_{034} - \bar{R}_{032} &= 0 \end{aligned} \quad (6)$$

$$\begin{aligned} \Sigma \bar{R}_{0(III, IV)} &= 0 \\ \bar{R}_{043} - \bar{R}_{045} &= 0 \end{aligned} \quad (7)$$

Substituting elastic weights (Table 2-3) into the elasto-static equations (Eqs. 1-7), the corresponding equations in terms of actual moments and geometry and section parameters, are obtained. These are shown as rows 2, 6, 7, 10, 11, 14, and 15 in the equilibrium matrix (Table 2-4).

### 2-3. Static Equations

There are no vertical displacements because the frame is assumed to be supported by a rigid foundation. Hence, two equations of

static equilibrium are considered at each joint

$$(1) \text{ Joint Moment Equation} \quad \Sigma M_j = 0$$

$$(2) \text{ Joint Force Equation} \quad \Sigma F_{jx} = 0$$

Ten equations of statics, five joint moment equations and five joint force equations, are available for this four-span frame.

$$\Sigma M_1 = 0 \quad M_{10} - M_{1A} = 0 \quad (8)$$

$$\Sigma M_2 = 0 \quad M_{2A} + M_{2B} - M_{20} = 0 \quad (9)$$

$$\Sigma M_3 = 0 \quad -M_{3B} - M_{3C} + M_{30} = 0 \quad (10)$$

$$\Sigma M_4 = 0 \quad M_{4C} + M_{4D} - M_{40} = 0 \quad (11)$$

$$\Sigma M_5 = 0 \quad -M_{50} + M_{5D} = 0 \quad (12)$$

$$\Sigma H_1 = 0 \quad -H_{1A} - H_{10} + P_1 = 0 \quad (13)$$

$$\Sigma H_2 = 0 \quad -H_{2A} - H_{20} - H_{2B} + P_2 = 0 \quad (14)$$

$$\Sigma H_3 = 0 \quad -H_{3B} - H_{30} - H_{3C} + P_3 = 0 \quad (15)$$

$$\Sigma H_4 = 0 \quad -H_{4C} - H_{40} - H_{4D} + P_4 = 0 \quad (16)$$

$$\Sigma H_5 = 0 \quad -H_{5D} - H_{50} + P_5 = 0 \quad (17)$$

Horizontal force components at the joints, shown in Fig. 2-4, are given as follows:

$$H_{jm} = \frac{1}{\beta L} \left( -\frac{M_{jm}}{2} + M_m - \frac{M_{im}}{2} - \frac{wL^2}{8} \right)$$

$$H_{jn} = \frac{1}{\beta L} \left( -\frac{M_{jn}}{2} + M_n - \frac{M_{kn}}{2} + \frac{wL^2}{8} \right)$$

$$H_{jo} = \frac{-M_{jo}}{\alpha L}$$

#### 2-4. Equilibrium Matrix

The equilibrium matrix, represented by seven elasto-static equations and ten static equations in terms of seventeen unknowns

(end moments), is presented in Table 2-4.

The resulting matrix can also be formed by successive addition of panels on the frame. Letting the one-span gable frame be a basic unit, a  $5 \times 5$  matrix, consisting of one elasto-static equation and four static equations, is obtained. If the frame has more than one panel (span), each panel introduces four additional unknowns and there are four additional equations available, two elasto-static equations and two static equations. Thus, there are always as many equations as unknowns. Table 2-4 is organized to illustrate this; thus, the equilibrium matrix for a one-, two-, three-, or four-span frame is represented. The matrices for the different frames are indicated by dashed lines. Figure 2-5 also shows the procedure of successive addition of panels to form frames having more spans, and the corresponding equations.

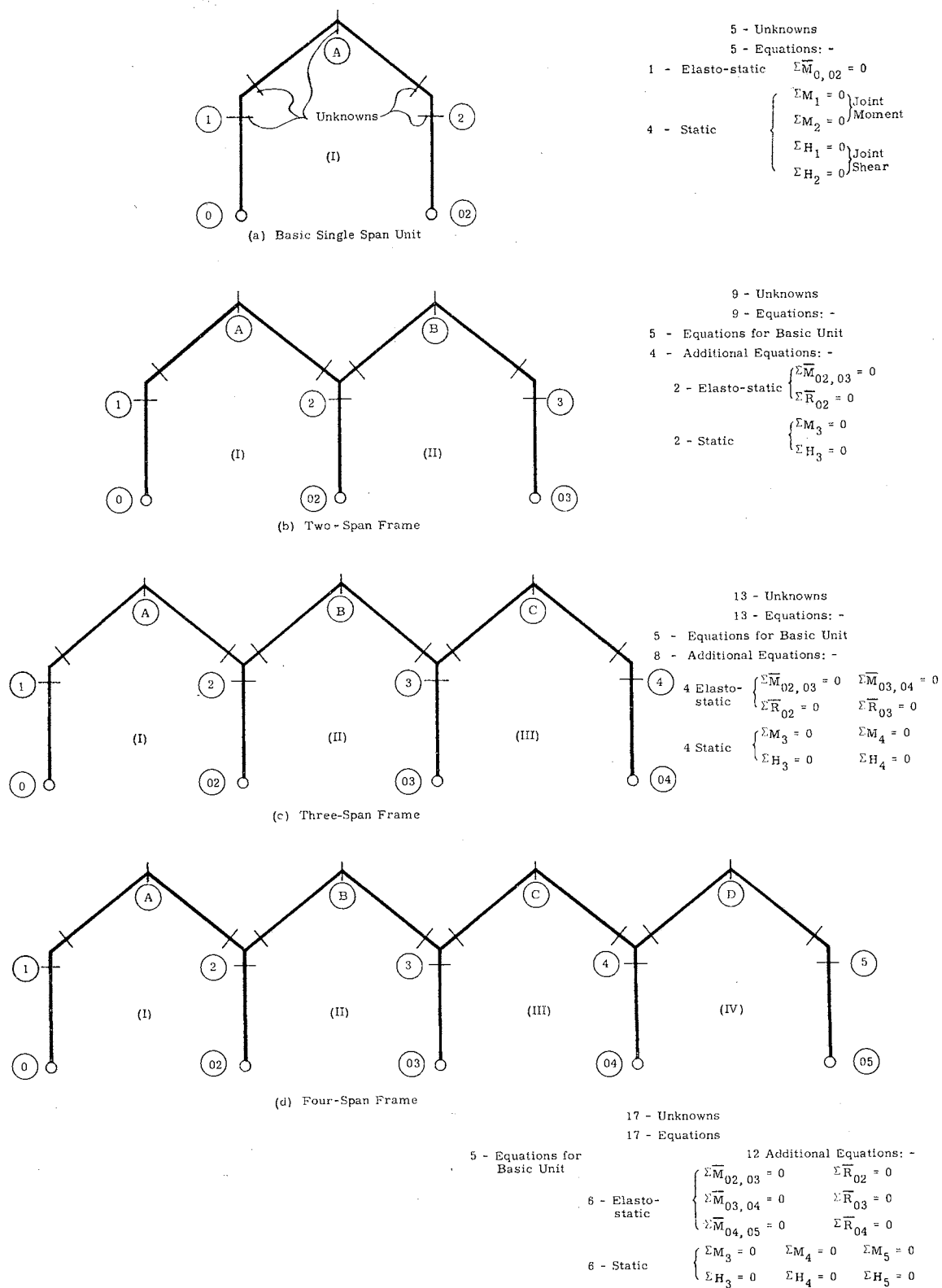


FIG. 2-5. FRAME FORMATION FROM BASIC SINGLE SPAN UNIT

TABLE 2-4. EQUILIBRIUM MATRIX

TABLE 2-4. EQUILIBRIUM MATRIX																											
M's Eq's	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23				
	M <sub>10</sub>	M <sub>1A</sub>	M <sub>A</sub>	M <sub>2A</sub>	M <sub>20</sub>	M <sub>2B</sub>	M <sub>B</sub>	M <sub>3B</sub>	M <sub>30</sub>	M <sub>3C</sub>	M <sub>C</sub>	M <sub>4C</sub>	M <sub>40</sub>	M <sub>4D</sub>	M <sub>D</sub>	M <sub>5D</sub>	M <sub>50</sub>	wL <sup>2</sup> Case 0	pl. Case 1	Case 2	Case 3	Case 4	Case 5				
1	1	-1																									
2	$\frac{\sigma^2}{3}$	$\frac{\sigma\nu}{4r_1} + \frac{\beta\nu}{12r_1}$	$\frac{\sigma\nu}{2r_1} + \frac{\beta\nu}{3r_1}$	$\frac{\sigma\nu}{4r_1} + \frac{\beta\nu}{12r_1}$	$\frac{\sigma^2}{3r_2}$	One-Span Matrix								* Zero for Two-, Three-, Four-, Span Matrices.				$\frac{\sigma\nu}{48r_1} + \frac{\beta\nu}{96r_1}$									
3	$-\frac{1}{\sigma}$	$-\frac{1}{2\beta}$	$\frac{1}{\beta}$	$-\frac{1}{2\beta}$						Two-Span Matrix									** Zero for Three-, Four-Span Matrices.				$\frac{1}{8\beta}$	-1			
4		$\frac{1}{2\beta}$	$-\frac{1}{\beta}$	$\frac{1}{2\beta}$	$\frac{1}{\sigma}$	$\frac{1}{2\beta}$	$-\frac{1}{\beta}$	$\frac{1}{2\beta}$											$(-\frac{1}{8\beta})^*$		-1						
5				1	-1	1																					
6					$\frac{\sigma^2}{3r_2}$	$\frac{\sigma\nu}{4r_3} + \frac{\beta\nu}{12r_3}$	$\frac{\sigma\nu}{2r_3} + \frac{\beta\nu}{3r_3}$	$\frac{\sigma\nu}{4r_3} + \frac{\beta\nu}{12r_3}$	$\frac{\sigma^2}{3r_4}$										$\frac{\sigma\nu}{48r_3} + \frac{\beta\nu}{96r_3}$								
7		$\frac{1}{6r_1}$	$\frac{1}{r_1}$	$\frac{5}{6r_1}$		$-\frac{5}{6r_3}$	$-\frac{1}{r_3}$	$-\frac{1}{6r_3}$											$\frac{\sigma\nu}{24r_1} + \frac{\beta\nu}{24r_3}$								
8						$-\frac{1}{2\beta}$	$\frac{1}{\beta}$	$-\frac{1}{2\beta}$	$-\frac{1}{\sigma}$	$-\frac{1}{2\beta}$	$\frac{1}{\beta}$	$-\frac{1}{2\beta}$	Three-Span Matrix					$(-\frac{1}{8\beta})^{**}$			-1						
9								-1	1	-1																	
10									$\frac{\sigma^2}{3r_4}$	$\frac{\sigma\nu}{4r_5} + \frac{\beta\nu}{12r_5}$	$\frac{\sigma\nu}{2r_5} + \frac{\beta\nu}{3r_5}$	$\frac{\sigma\nu}{4r_5} + \frac{\beta\nu}{12r_5}$	$\frac{\sigma^2}{3r_6}$					$\frac{\sigma\nu}{48r_5} + \frac{\beta\nu}{96r_5}$									
11						$-\frac{1}{6r_3}$	$-\frac{1}{r_3}$	$-\frac{5}{6r_3}$		$\frac{5}{6r_5}$	$\frac{1}{r_5}$	$\frac{1}{6r_5}$						$\frac{\sigma\nu}{24r_3} + \frac{\beta\nu}{24r_5}$									
12										$\frac{1}{2\beta}$	$-\frac{1}{\beta}$	$\frac{1}{2\beta}$	$\frac{1}{\sigma}$	$\frac{1}{2\beta}$	$-\frac{1}{\beta}$	$\frac{1}{2\beta}$		$(-\frac{1}{8\beta})^{***}$				-1					
13												1	-1	1													
14													$\frac{\sigma^2}{3r_6}$	$\frac{\sigma\nu}{4r_7} + \frac{\beta\nu}{12r_7}$	$\frac{\sigma\nu}{2r_7} + \frac{\beta\nu}{3r_7}$	$\frac{\sigma\nu}{4r_7} + \frac{\beta\nu}{12r_7}$	$\frac{\sigma^2}{3r_8}$	$\frac{\sigma\nu}{48r_7} + \frac{\beta\nu}{96r_7}$									
15										$\frac{1}{6r_5}$	$\frac{1}{r_5}$	$\frac{1}{6r_5}$		$-\frac{5}{6r_7}$	$-\frac{1}{r_7}$	$-\frac{1}{6r_7}$		$\frac{1}{24r_5} + \frac{1}{24r_7}$									
16														$-\frac{1}{2\beta}$	$\frac{1}{2\beta}$	$-\frac{1}{2\beta}$	$-\frac{1}{\sigma}$	$-\frac{1}{8\beta}$					-1				
17																-1	1	Four-Span Matrix									

## CHAPTER III

### COMPUTER PROGRAM

#### 3-1. General

The program is a precise sequence of coded instructions which an electronic computer interprets to solve a particular problem. The programming of any problem on an electronic computer is accomplished in two steps: preparation of the flow chart (flow diagram) which is a "road-map" of the procedure of solution for a given problem, and preparation of the program which is written on a coding form. The FORTRAN language has been adopted for use in writing this program. A general computer program for determination of end moments in accordance with the matrix obtained in Chapter II (Table 2-4) is developed. Crout's method is selected for solving the set of simultaneous equations necessary to obtain final moments. Crout's method is briefly explained in the next section.

#### 3-2. Crout's Method (16, 17)

Consider a set of five linear equations with five unknowns:

$$A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + A_{14}X_4 + A_{15}X_5 = C_1 \quad (1)$$

$$A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + A_{24}X_4 + A_{25}X_5 = C_2 \quad (2)$$

$$A_{31}X_1 + A_{32}X_2 + A_{33}X_3 + A_{34}X_4 + A_{35}X_5 = C_3 \quad (3)$$

$$A_{41}X_1 + A_{42}X_2 + A_{43}X_3 + A_{44}X_4 + A_{45}X_5 = C_4 \quad (4)$$

$$A_{51}X_1 + A_{52}X_2 + A_{53}X_3 + A_{54}X_4 + A_{55}X_5 = C_5 \quad (5)$$

these equations can be expressed in a compact matrix form as shown in Fig. 3-1a; two auxiliary triangular matrices, (Fig. 3-1b), called the G- and H-matrices, are obtained by the following relationship.

$$\begin{bmatrix} G \end{bmatrix} \begin{bmatrix} H \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}$$

Thus, the elements of these auxiliary matrices are obtained as

$$\begin{array}{ll} G_{11} = A_{11} & H_{12} = \frac{A_{12}}{A_{11}} \\ G_{21} = A_{21} & H_{13} = \frac{A_{13}}{A_{11}} \\ \text{-----} & \text{-----} \\ G_{22} = A_{22} - G_{21} \times H_{12} & H_{23} = \frac{A_{23} - G_{21} \times H_{13}}{G_{22}} \\ G_{32} = A_{32} - G_{31} \times H_{12} & H_{24} = \frac{A_{24} - G_{21} \times H_{14}}{G_{22}} \\ \text{-----} & \text{-----} \\ G_{33} = A_{33} - G_{31} \times H_{13} - G_{32} \times H_{23} & H_{34} = \frac{A_{34} - G_{31} \times H_{14} - G_{32} \times H_{24}}{G_{33}} \\ G_{43} = A_{43} - G_{41} \times H_{13} - G_{42} \times H_{23} & H_{35} = \frac{A_{35} - G_{31} \times H_{15} - G_{32} \times H_{25}}{G_{33}} \\ \text{-----} & \text{-----} \end{array}$$

or, in compact form

$$G_{IJ} = A_{IJ} - \sum_{LL} (G_{I, LL} \times H_{LL, J}); \quad H_{IJ} = \frac{A_{IJ} - \sum_L (G_{I, L} \times H_{LJ})}{G_{II}}$$

Once the auxiliary matrices have been determined, the solution (unknowns) can be calculated directly as

$$X_5 = H_{56}$$

$$X_4 = H_{46} - H_{45} \times X_5$$

$$X_3 = H_{36} - H_{35} \times X_5 - H_{34} \times X_4$$

or, in compact form

$$X_I = H_{I,KK} - \sum H_{I,K} \times X_K$$

$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$C$
$A_{11}$	$A_{12}$	$A_{13}$	$A_{14}$	$A_{15}$	$A_{16}$
$A_{21}$	$A_{22}$	$A_{23}$	$A_{24}$	$A_{25}$	$A_{26}$
$A_{31}$	$A_{32}$	$A_{33}$	$A_{34}$	$A_{35}$	$A_{36}$
$A_{41}$	$A_{42}$	$A_{43}$	$A_{44}$	$A_{45}$	$A_{46}$
$A_{51}$	$A_{52}$	$A_{53}$	$A_{54}$	$A_{55}$	$A_{56}$

(a) Original Matrix

$G_{11}$	$H_{12}$	$H_{13}$	$H_{14}$	$H_{15}$	$H_{16}$
$G_{21}$	$G_{22}$	$H_{23}$	$H_{24}$	$H_{35}$	$H_{36}$
$G_{31}$	$G_{32}$	$G_{33}$	$H_{34}$	$H_{35}$	$H_{36}$
$G_{41}$	$G_{42}$	$G_{43}$	$G_{44}$	$H_{45}$	$H_{46}$
$G_{51}$	$G_{52}$	$G_{53}$	$G_{54}$	$G_{55}$	$H_{56}$

(b) Auxiliary Matrices

FIG. 3-1. MATRICES



### 3-3. Flow Chart

The flow chart shown in Fig. 3-2 was prepared as an aid to setting up the sequence of instructions for computation of end moments. The set of symbols appearing on the flow chart have been selected for use with the FORTRAN language; statement numbers are shown on the upper left corner of some blocks.

### 3-4. Statement of Program

The program has been written in IBM 1620 FORTRAN. Floating decimal arithmetic is used; the subroutine of SQR(x) which is incorporated in this program is available at the Engineering Computer Laboratory of Oklahoma State University.

FORTRAN statements are presented in Table 3-1. The program is prepared for use in computing the end moments of a specific frame, or in computing end moment coefficients for specified ranges of the parameters  $\alpha$ ,  $\beta$ , and  $r$ 's. The combinations of parameters  $\alpha$ ,  $\beta$ , and  $r$ 's believed to be most practical are as follow:

$$\alpha = 0.1 (0.1) 1.0$$

$$\beta = 0.1 (0.1) 0.5$$

$$r_1 = 0.6 (0.2) 1.4$$

$$r_2 = 0.8 (0.2) 1.2$$

$$r_3 = 0.6 (0.2) 1.4$$

$$r_4 = 0.8 (0.2) 1.2$$

$$r_5 = 0.6 (0.2) 1.4$$

$$r_6 = 0.8 (0.2) 1.2$$

$$r_7 = 0.6 (0.2) 1.4$$

$$r_8 = 0.8 (0.2) 1.2$$

Intermediate results should be obtainable by linear interpolation.

### 3-5. Input Data

Six input data cards are required. The first card contains seven words which completely describe the frame and the type of

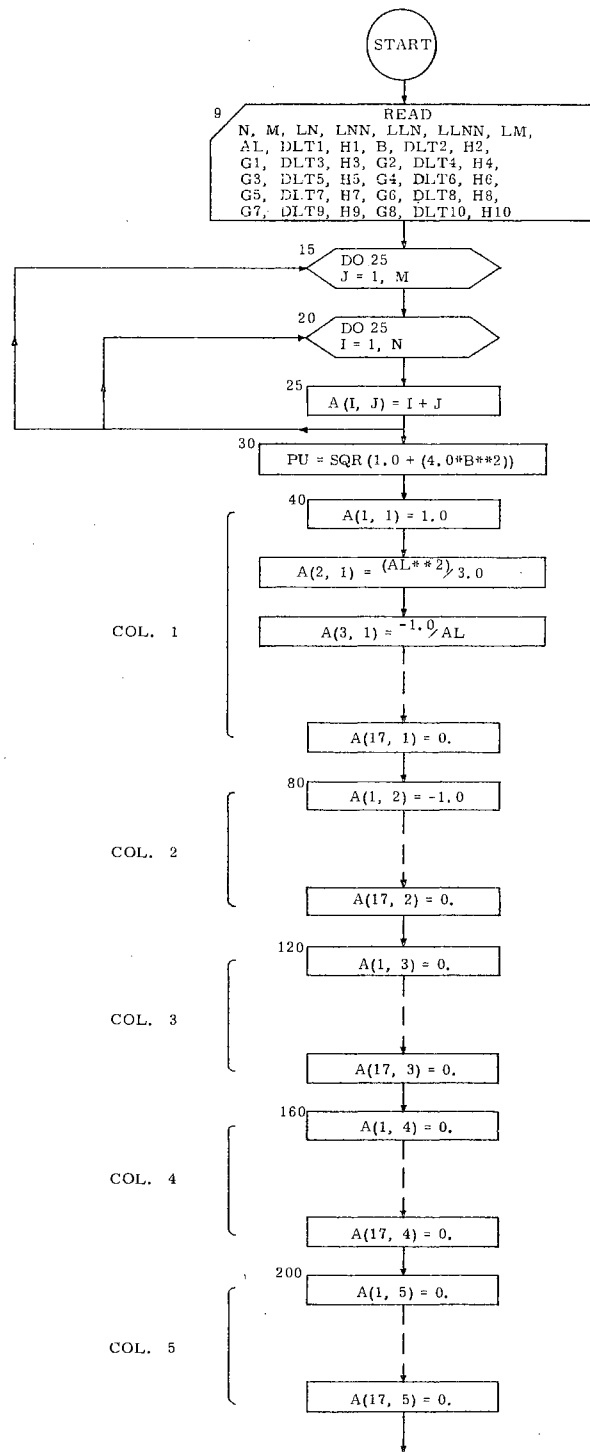


FIG. 3-2. FLOW CHART

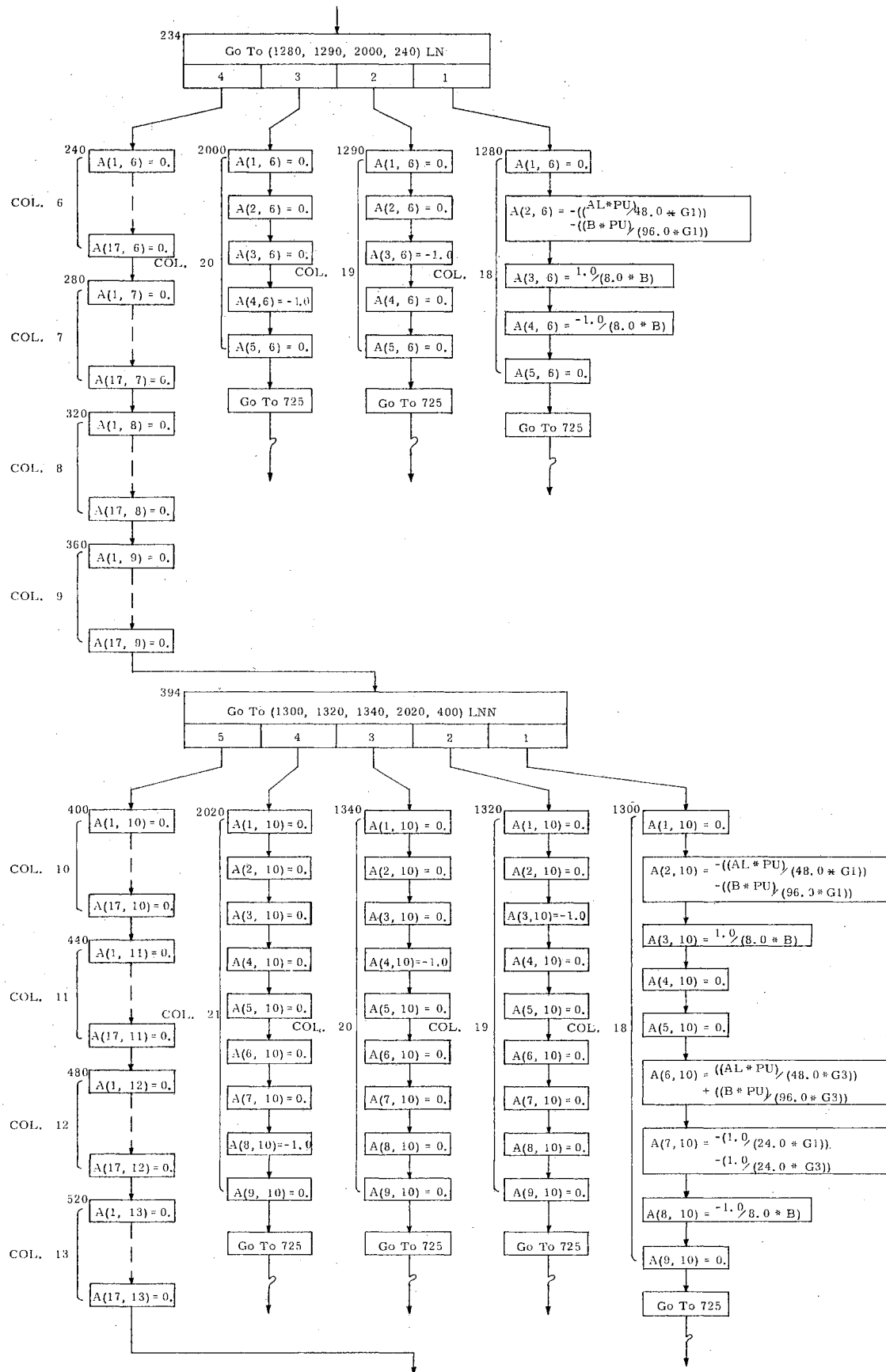


FIG. 3-2. (Continued)

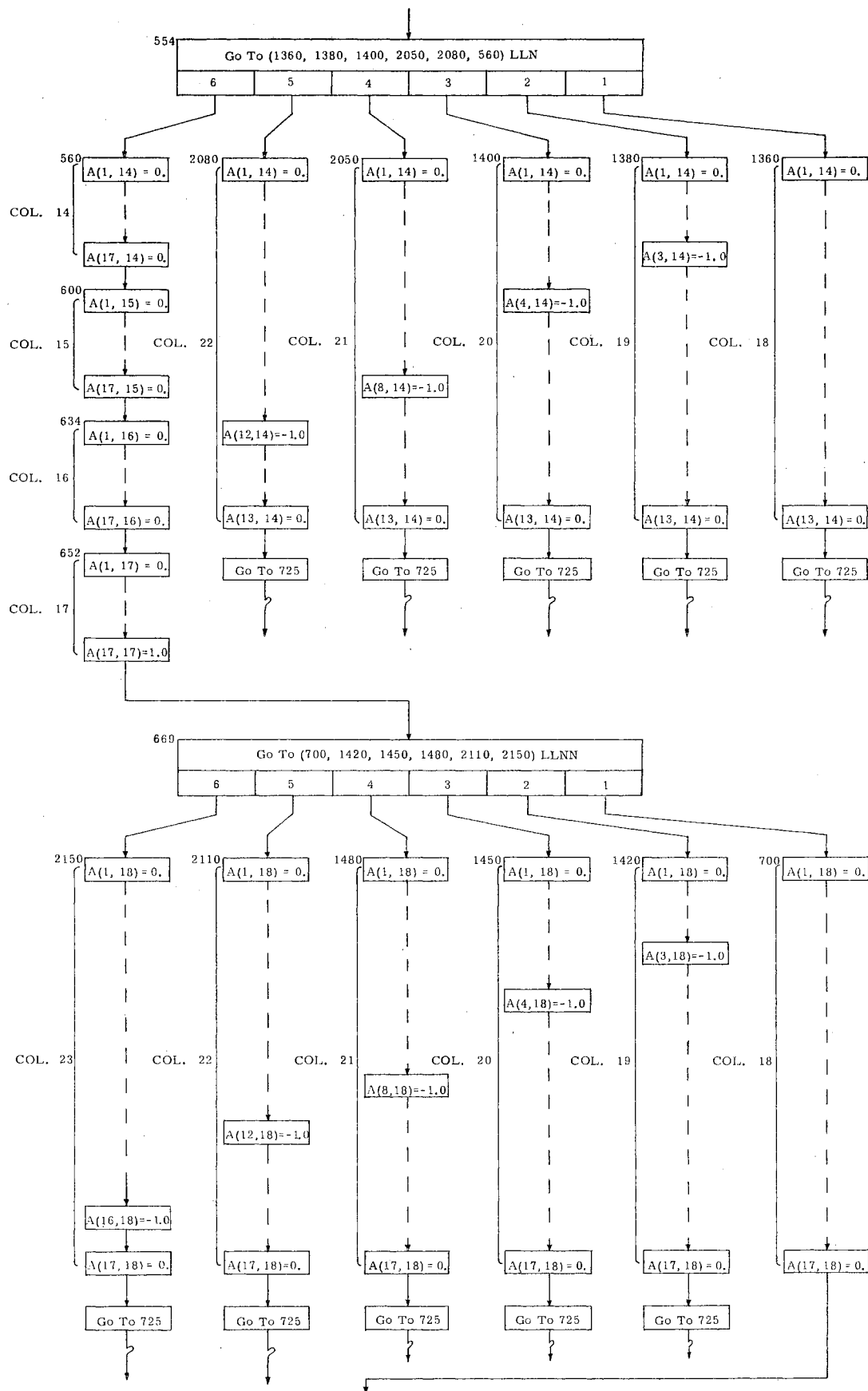


FIG. 3-2. (Continued)

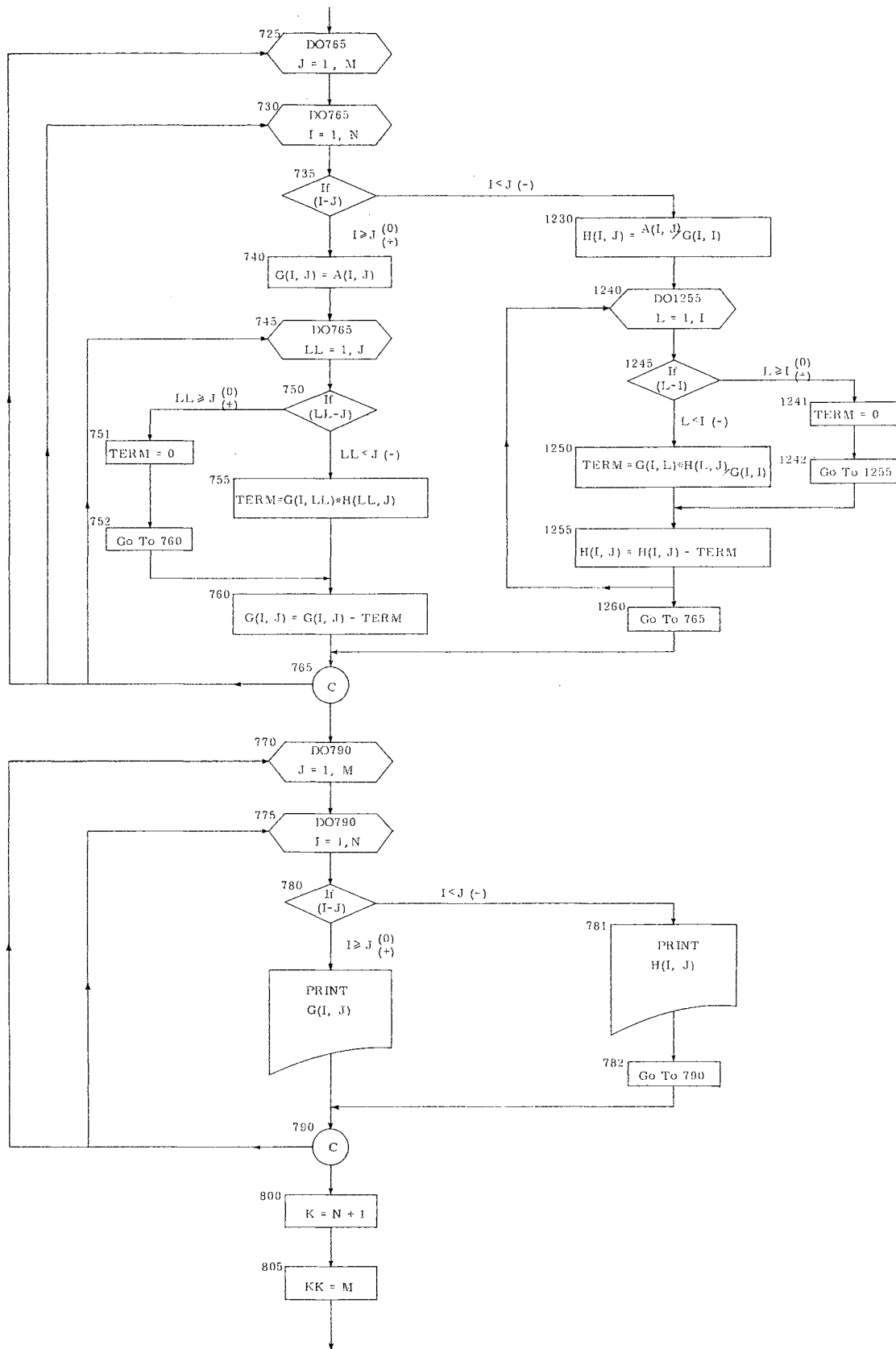


FIG. 3-2. (Continued)

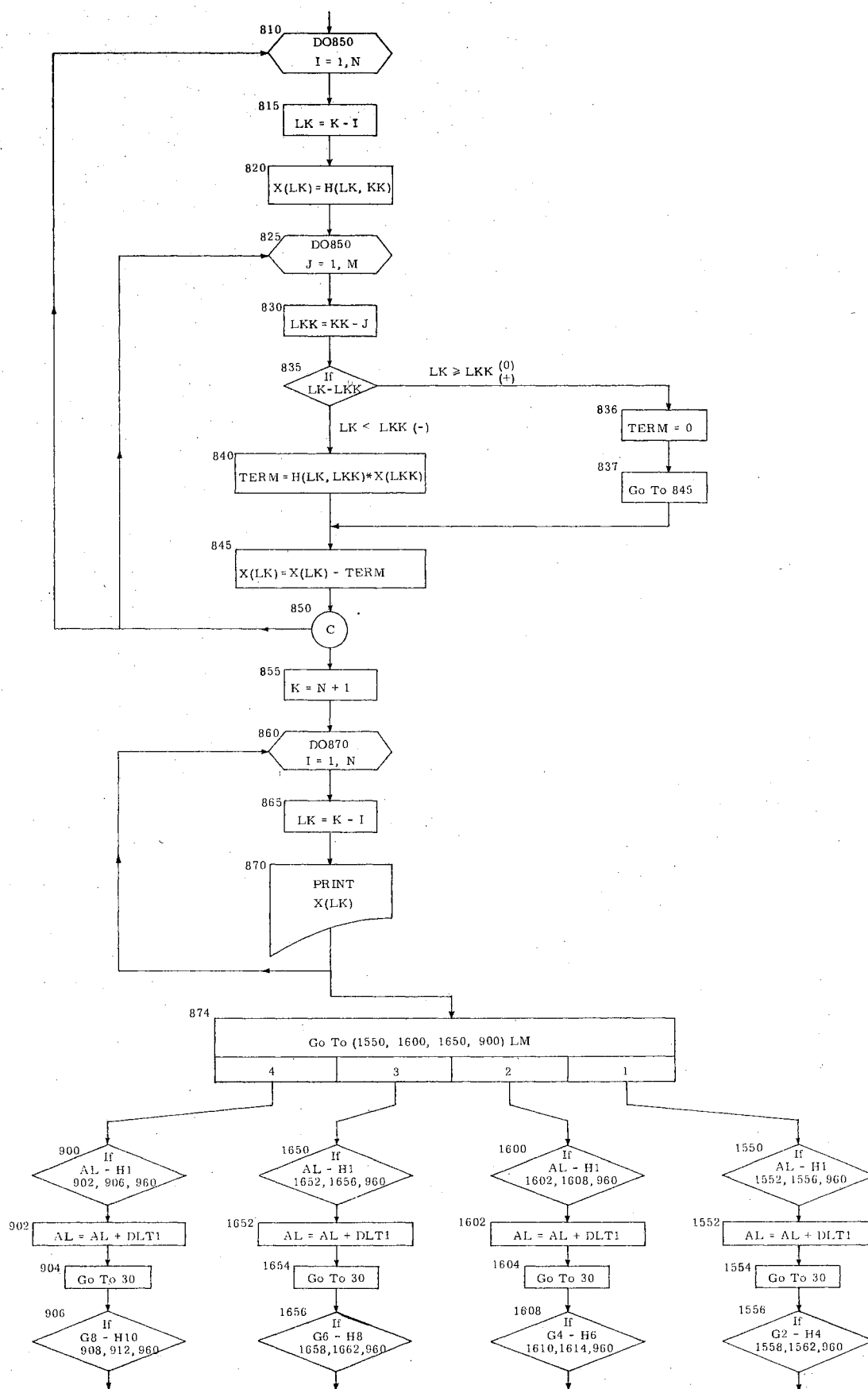


FIG. 3-2. (Continued)

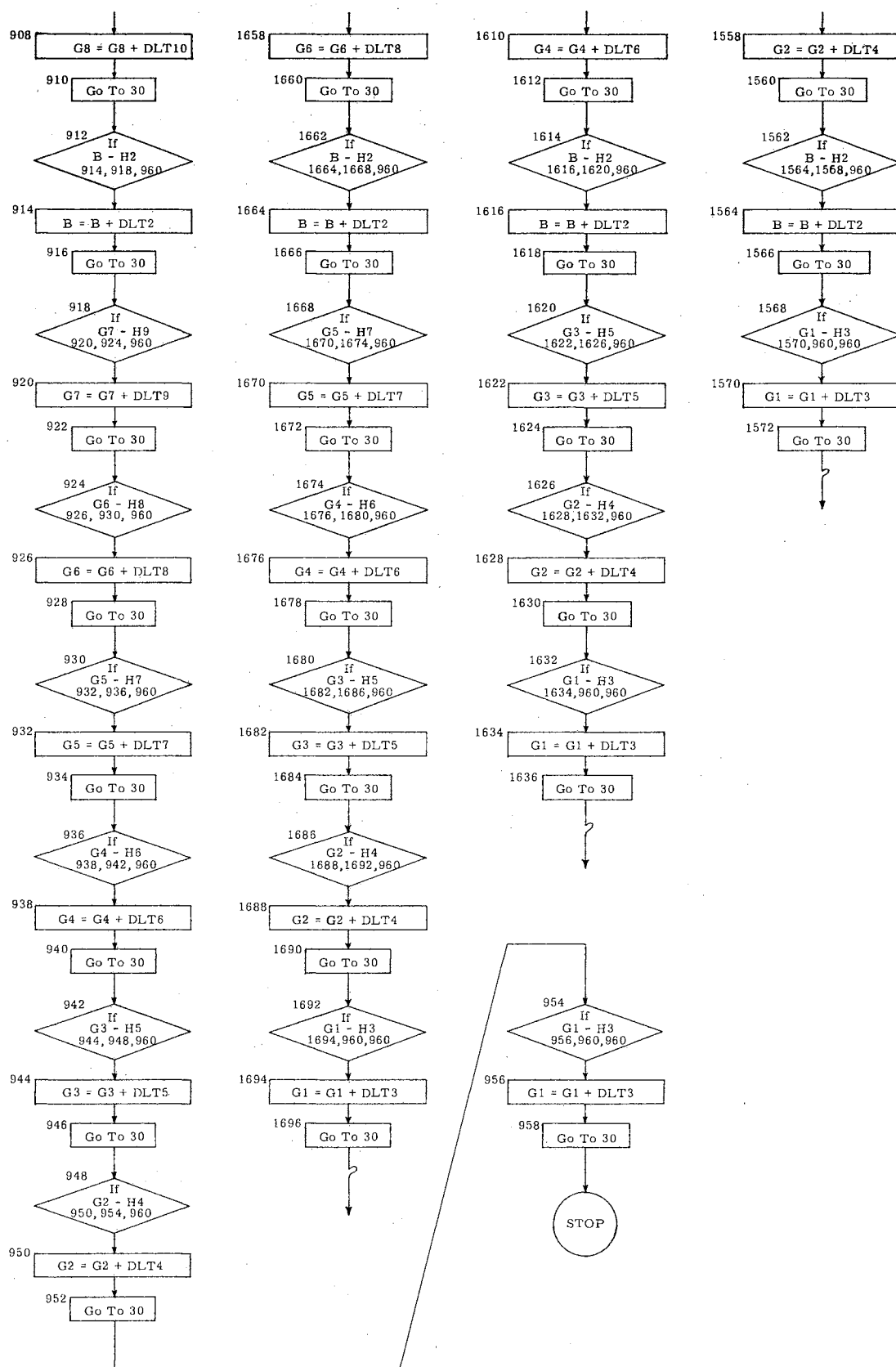


FIG. 3-2. (Continued)

TABLE 3-1. FORTRAN STATEMENTS

```

C      END MOMENTS FOR ONE, TWO, THREE AND FOUR GABLE FRAMES
C      DUE TO UNIFORM LOAD AND UNIT FORCE

1      DIMENSION A(17,18),G(17,18),H(17,18),X(17)
9      READ 400, N,M,LN,LNN,LLN,LLNN,LM
10     READ 402, AL,DLT1,H1,B,DLT2,H2
11     READ 402, G1,DLT3,H3,G2,DLT4,H4
12     READ 402, G3,DLT5,H5,G4,DLT6,H6
13     READ 402, G5,DLT7,H7,G6,DLT8,H8
14     READ 402, G7,DLT9,H9,G8,DLT10,H10
15     DO 25 J=1,M
20     DO 25 I=1,N
25     A(I,J) = I+J
30     PU = SQRT(1.0+(4.0*B**2))
40     A(1,1) = 1.0
42     A(2,1) = (AL**2)/3.0
44     A(3,1) = -1.0/AL
46     A(4,1) = 0.
48     A(5,1) = 0.
50     A(6,1) = 0.
52     A(7,1) = 0.
54     A(8,1) = 0.
56     A(9,1) = 0.
58     A(10,1) = 0.
60     A(11,1) = 0.
62     A(12,1) = 0.
64     A(13,1) = 0.
66     A(14,1) = 0.
68     A(15,1) = 0.
70     A(16,1) = 0.
72     A(17,1) = 0.
80     A(1,2) = -1.0
82     A(2,2) = ((AL*PU)/(4.0*G1))+((B*PU)/(12.0*G1))
84     A(3,2) = -1.0/(2.0*B)
86     A(4,2) = 1.0/(2.0*B)
88     A(5,2) = 0.
90     A(6,2) = 0.
92     A(7,2) = 1.0/(6.0*G1)
94     A(8,2) = 0.
96     A(9,2) = 0.
98     A(10,2) = 0.
100    A(11,2) = 0.
102    A(12,2) = 0.
104    A(13,2) = 0.
106    A(14,2) = 0.
108    A(15,2) = 0.
110    A(16,2) = 0.
112    A(17,2) = 0.
120    A(1,3) = 0.
122    A(2,3) = ((AL*PU)/(2.0*G1))+((B*PU)/(3.0*G1))
124    A(3,3) = 1.0/B
126    A(4,3) = -1.0/B
128    A(5,3) = 0.
130    A(6,3) = 0.
132    A(7,3) = 1.0/G1
134    A(8,3) = 0.
136    A(9,3) = 0.
138    A(10,3) = 0.
140    A(11,3) = 0.
142    A(12,3) = 0.
144    A(13,3) = 0.
146    A(14,3) = 0.
148    A(15,3) = 0.
150    A(16,3) = 0.
152    A(17,3) = 0.
160    A(1,4) = 0.
162    A(2,4) = ((AL*PU)/(4.0*G1))+((B*PU)/(12.0*G1))
164    A(3,4) = -1.0/(2.0*B)
166    A(4,4) = 1.0/(2.0*B)
168    A(5,4) = 0.
170    A(6,4) = 0.
172    A(7,4) = 5.0/(6.0*G1)
174    A(8,4) = 0.
176    A(9,4) = 0.
178    A(10,4) = 0.
180    A(11,4) = 0.
182    A(12,4) = 0.
184    A(13,4) = 0.
186    A(14,4) = 0.
188    A(15,4) = 0.
190    A(16,4) = 0.
192    A(17,4) = 0.
200    A(1,5) = 0.
202    A(2,5) = (AL**2)/(3.0*G2)
204    A(3,5) = 0.
206    A(4,5) = 1.0/AL
208    A(5,5) = -1.0
210    A(6,5) = (AL**2)/(3.0*G2)
212    A(7,5) = 0.
214    A(8,5) = 0.
216    A(9,5) = 0.
218    A(10,5) = 0.
220    A(11,5) = 0.
222    A(12,5) = 0.
224    A(13,5) = 0.
226    A(14,5) = 0.
228    A(15,5) = 0.
230    A(16,5) = 0.
232    A(17,5) = 0.

234    GO TO(1280,1290,2000,240)LN
240    A(1,6) = 0.
242    A(2,6) = 0.
244    A(3,6) = 0.
246    A(4,6) = 1.0/(2.0*B)
248    A(5,6) = 1.0
250    A(6,6) = ((AL*PU)/(4.0*G3))+((B*PU)/(12.0*G3))
252    A(7,6) = -5.0/(6.0*G3)
254    A(8,6) = -1.0/(2.0*B)
256    A(9,6) = 0.
258    A(10,6) = 0.
260    A(11,6) = -1.0/(6.0*G3)
262    A(12,6) = 0.
264    A(13,6) = 0.
266    A(14,6) = 0.
268    A(15,6) = 0.
270    A(16,6) = 0.
272    A(17,6) = 0.
280    A(1,7) = 0.
282    A(2,7) = 0.
284    A(3,7) = 0.
286    A(4,7) = -1.0/B
288    A(5,7) = 0.
290    A(6,7) = ((AL*PU)/(2.0*G3))+((B*PU)/(3.0*G3))
292    A(7,7) = -1.0/G3
294    A(8,7) = 1.0/B
296    A(9,7) = 0.
298    A(10,7) = 0.
300    A(11,7) = -1.0/G3
302    A(12,7) = 0.
304    A(13,7) = 0.
306    A(14,7) = 0.
308    A(15,7) = 0.
310    A(16,7) = 0.
312    A(17,7) = 0.
320    A(1,8) = 0.
322    A(2,8) = 0.
324    A(3,8) = 0.
326    A(4,8) = 1.0/(2.0*B)
328    A(5,8) = 0.
330    A(6,8) = ((AL*PU)/(4.0*G3))+((B*PU)/(12.0*G3))
332    A(7,8) = -1.0/(6.0*G3)
334    A(8,8) = -1.0/(2.0*B)
336    A(9,8) = -1.0
338    A(10,8) = 0.
340    A(11,8) = -5.0/(6.0*G3)
342    A(12,8) = 0.
344    A(13,8) = 0.
346    A(14,8) = 0.
348    A(15,8) = 0.
350    A(16,8) = 0.
352    A(17,8) = 0.
360    A(1,9) = 0.
362    A(2,9) = 0.
364    A(3,9) = 0.
366    A(4,9) = 0.
368    A(5,9) = 0.
370    A(6,9) = (AL**2)/(3.0*G4)
372    A(7,9) = 0.
374    A(8,9) = -1.0/AL
376    A(9,9) = 1.0
378    A(10,9) = (AL**2)/(3.0*G4)
380    A(11,9) = 0.
382    A(12,9) = 0.
384    A(13,9) = 0.
386    A(14,9) = 0.
388    A(15,9) = 0.
390    A(16,9) = 0.
392    A(17,9) = 0.
394    GO TO(1300,1320,1340,2020,400)LNN
400    A(1,10) = 0.
402    A(2,10) = 0.
404    A(3,10) = 0.
406    A(4,10) = 0.
408    A(5,10) = 0.
410    A(6,10) = 0.
412    A(7,10) = 0.
414    A(8,10) = -1.0/(2.0*B)
416    A(9,10) = -1.0
418    A(10,10) = ((AL*PU)/(4.0*G5))+((B*PU)/(12.0*G5))
420    A(11,10) = 5.0/(6.0*G5)
422    A(12,10) = 1.0/(2.0*B)
424    A(13,10) = 0.
426    A(14,10) = 0.
428    A(15,10) = 1.0/(6.0*G5)
430    A(16,10) = 0.
432    A(17,10) = 0.
440    A(1,11) = 0.
442    A(2,11) = 0.
444    A(3,11) = 0.
446    A(4,11) = 0.
448    A(5,11) = 0.
450    A(6,11) = 0.
452    A(7,11) = 0.
454    A(8,11) = 1.0/B
456    A(9,11) = 0.
458    A(10,11) = ((AL*PU)/(2.0*G5))+((B*PU)/(3.0*G5))
460    A(11,11) = 1.0/G5
462    A(12,11) = -1.0/B

```



TABLE 3-1 (Continued)

464	A(13,11) = 0.	657	A(6,17) = 0.
466	A(14,11) = 0.	658	A(7,17) = 0.
468	A(15,11) = 1.0/G5	659	A(8,17) = 0.
470	A(16,11) = 0.	660	A(9,17) = 0.
472	A(17,11) = 0.	661	A(10,17) = 0.
480	A(1,12) = 0.	662	A(11,17) = 0.
482	A(2,12) = 0.	663	A(12,17) = 0.
484	A(3,12) = 0.	664	A(13,17) = 0.
486	A(4,12) = 0.	665	A(14,17) = (AL**2)/(3.0*G8)
488	A(5,12) = 0.	666	A(15,17) = 0.
497	A(6,12) = 0.	667	A(16,17) = -1.0/AL
492	A(7,12) = 0.	668	A(17,17) = 1.0
494	A(8,12) = -1.0/(2.0*B)	669	GO TO (700,1420,1450,1480,2110,2150)LLNN
496	A(9,12) = 0.	700	A(1,18) = 0.
498	A(10,12) = ((AL*PU)/(4.0*G5))+((B*PU)/(12.0*G5))	701	A(2,18) = -((AL*PU)/(48.0*G1))-((B*PU)/(96.0*G1))
510	A(11,12) = 1.0/(6.0*G5)	702	A(3,18) = 1.0/(8.0*B)
502	A(12,12) = 1.0/(2.0*B)	703	A(4,18) = 0.
504	A(13,12) = 1.0	704	A(5,18) = 0.
506	A(14,12) = 0.	705	A(6,18) = ((AL*PU)/(48.0*G3))+((B*PU)/(96.0*G3))
508	A(15,12) = 5.0/(6.0*G5)	706	A(7,18) = -1.0/(24.0*G1)-(1.0/(24.0*G3))
510	A(16,12) = 0.	707	A(8,18) = 0.
512	A(17,12) = 0.	708	A(9,18) = 0.
520	A(1,13) = 0.	709	A(10,18) = -((AL*PU)/(48.0*G5))-((B*PU)/(96.0*G5))
522	A(2,13) = 0.	710	A(11,18) = -1.0/(24.0*G3)-(1.0/(24.0*G5))
524	A(3,13) = 0.	711	A(12,18) = 0.
526	A(4,13) = 0.	712	A(13,18) = 0.
528	A(5,13) = 0.	713	A(14,18) = ((AL*PU)/(48.0*G7))+((B*PU)/(96.0*G7))
530	A(6,13) = 0.	714	A(15,18) = -1.0/(24.0*G5)-(1.0/(24.0*G7))
532	A(7,13) = 0.	715	A(16,18) = -1.0/(8.0*B)
534	A(8,13) = 0.	716	A(17,18) = 0.
536	A(9,13) = 0.	717	GO TO 725
538	A(10,13) = (AL**2)/(3.0*G6)	1280	A(1,6) = 0.
540	A(11,13) = 0.	1281	A(2,6) = -((AL*PU)/(48.0*G1))-((B*PU)/(96.0*G1))
542	A(12,13) = 1.0/AL	1282	A(3,6) = 1.0/(8.0*B)
544	A(13,13) = -1.0	1283	A(4,6) = -1.0/(8.0*B)
546	A(14,13) = (AL**2)/(3.0*G6)	1284	A(5,6) = 0.
548	A(15,13) = 0.	1285	GO TO 725
550	A(16,13) = 0.	1290	A(1,6) = 0.
552	A(17,13) = 0.	1291	A(2,6) = 0.
554	GO TO (1360,1380,1400,2050,2080,560)LLN	1292	A(3,6) = -1.0
560	A(1,14) = 0.	1293	A(4,6) = 0.
562	A(2,14) = 0.	1294	A(5,6) = 0.
564	A(3,14) = 0.	1295	GO TO 725
566	A(4,14) = 0.	2000	A(1,6) = 0.
568	A(5,14) = 0.	2002	A(2,6) = 0.
570	A(6,14) = 0.	2004	A(3,6) = 0.
572	A(7,14) = 0.	2008	A(4,6) = -1.0
574	A(8,14) = 0.	2010	A(5,6) = 0.
576	A(9,14) = 0.	2012	GO TO 725
578	A(10,14) = 0.	1300	A(1,10) = 0.
580	A(11,14) = 0.	1301	A(2,10) = -((AL*PU)/(48.0*G1))-((B*PU)/(96.0*G1))
582	A(12,14) = 1.0/(2.0*B)	1302	A(3,10) = 1.0/(8.0*B)
584	A(13,14) = 1.0	1303	A(4,10) = 0.
586	A(14,14) = ((AL*PU)/(4.0*G7))+((B*PU)/(12.0*G7))	1304	A(5,10) = 0.
588	A(15,14) = -5.0/(6.0*G7)	1305	A(6,10) = ((AL*PU)/(48.0*G3))+((B*PU)/(96.0*G3))
590	A(16,14) = -1.0/(2.0*B)	1306	A(7,10) = -1.0/(24.0*G1)-(1.0/(24.0*G3))
592	A(17,14) = 0.	1307	A(8,10) = -1.0/(8.0*B)
600	A(1,15) = 0.	1308	A(9,10) = 0.
602	A(2,15) = 0.	1309	GO TO 725
604	A(3,15) = 0.	1320	A(1,10) = 0.
606	A(4,15) = 0.	1321	A(2,10) = 0.
608	A(5,15) = 0.	1322	A(3,10) = -1.0
610	A(6,15) = 0.	1323	A(4,10) = 0.
612	A(7,15) = 0.	1324	A(5,10) = 0.
614	A(8,15) = 0.	1325	A(6,10) = 0.
616	A(9,15) = 0.	1326	A(7,10) = 0.
618	A(10,15) = 0.	1327	A(8,10) = 0.
620	A(11,15) = 0.	1328	A(9,10) = 0.
622	A(12,15) = -1.0/B	1329	GO TO 725
624	A(13,15) = 0.	1340	A(1,10) = 0.
626	A(14,15) = ((AL*PU)/(2.0*G7))+((B*PU)/(3.0*G7))	1341	A(2,10) = 0.
628	A(15,15) = -1.0/G7	1342	A(3,10) = 0.
630	A(16,15) = 1.0/(2.0*B)	1343	A(4,10) = -1.0
632	A(17,15) = 0.	1344	A(5,10) = 0.
634	A(1,16) = 0.	1345	A(6,10) = 0.
635	A(2,16) = 0.	1346	A(7,10) = 0.
636	A(3,16) = 0.	1347	A(8,10) = 0.
637	A(4,16) = 0.	1348	A(9,10) = 0.
638	A(5,16) = 0.	1349	GO TO 725
639	A(6,16) = 0.	2020	A(1,10) = 0.
640	A(7,16) = 0.	2022	A(2,10) = 0.
641	A(8,16) = 0.	2024	A(3,10) = 0.
642	A(9,16) = 0.	2026	A(4,10) = 0.
643	A(10,16) = 0.	2028	A(5,10) = 0.
644	A(11,16) = 0.	2030	A(6,10) = 0.
645	A(12,16) = 1.0/(2.0*B)	2032	A(7,10) = 0.
646	A(13,16) = 0.	2034	A(8,10) = -1.0
647	A(14,16) = ((AL*PU)/(4.0*G7))+((B*PU)/(12.0*G7))	2036	A(9,10) = 0.
648	A(15,16) = -1.0/(6.0*G7)	2038	GO TO 725
649	A(16,16) = -1.0/(2.0*B)	1360	A(1,14) = 0.
650	A(17,16) = -1.0	1361	A(2,14) = -((AL*PU)/(48.0*G1))-((B*PU)/(96.0*G1))
652	A(1,17) = 0.	1362	A(3,14) = 1.0/(8.0*B)
653	A(2,17) = 0.	1363	A(4,14) = 0.
654	A(3,17) = 0.	1364	A(5,14) = 0.
655	A(4,17) = 0.	1365	A(6,14) = ((AL*PU)/(48.0*G3))+((B*PU)/(96.0*G3))
656	A(5,17) = 0.	1366	A(7,14) = -1.0/(24.0*G1)-(1.0/(24.0*G3))

TABLE 3-1 (Continued)

1367	A(8,14) = 0.	1483	A(4,18) = 0.
1368	A(9,14) = 0.	1484	A(5,18) = 0.
1369	A(10,14) = -((AL*PU)/(48.0*G5))-((B*PU)/(96.0*G5))	1485	A(6,18) = 0.
1370	A(11,14) = -(1.0/(24.0*G3))-(1.0/(24.0*G5))	1486	A(7,18) = 0.
1371	A(12,14) = -1.0/(8.0*B)	1487	A(8,18) = -1.0
1372	A(13,14) = 0.	1488	A(9,18) = 0.
1373	GO TO 725	1489	A(10,18) = 0.
1380	A(1,14) = 0.	1490	A(11,18) = 0.
1381	A(2,14) = 0.	1491	A(12,18) = 0.
1382	A(3,14) = -1.0	1492	A(13,18) = 0.
1383	A(4,14) = 0.	1493	A(14,18) = 0.
1384	A(5,14) = 0.	1494	A(15,18) = 0.
1385	A(6,14) = 0.	1495	A(16,18) = 0.
1386	A(7,14) = 0.	1496	A(17,18) = 0.
1387	A(8,14) = 0.	1497	GO TO 725
1388	A(9,14) = 0.	2110	A(1,18) = 0.
1389	A(10,14) = 0.	2112	A(2,18) = 0.
1390	A(11,14) = 0.	2114	A(3,18) = 0.
1391	A(12,14) = 0.	2116	A(4,18) = 0.
1392	A(13,14) = 0.	2118	A(5,18) = 0.
1393	GO TO 725	2120	A(6,18) = 0.
1400	A(1,14) = 0.	2122	A(7,18) = 0.
1401	A(2,14) = 0.	2124	A(8,18) = 0.
1402	A(3,14) = 0.	2126	A(9,18) = 0.
1403	A(4,14) = -1.0	2128	A(10,18) = 0.
1404	A(5,14) = 0.	2130	A(11,18) = 0.
1405	A(6,14) = 0.	2132	A(12,18) = -1.0
1406	A(7,14) = 0.	2134	A(13,18) = 0.
1407	A(8,14) = 0.	2136	A(14,18) = 0.
1408	A(9,14) = 0.	2138	A(15,18) = 0.
1409	A(10,14) = 0.	2140	A(16,18) = 0.
1410	A(11,14) = 0.	2142	A(17,18) = 0.
1411	A(12,14) = 0.	2144	GO TO 725
1412	A(13,14) = 0.	2150	A(1,18) = 0.
1413	GO TO 725	2152	A(2,18) = 0.
2050	A(1,14) = 0.	2154	A(3,18) = 0.
2052	A(2,14) = 0.	2156	A(4,18) = 0.
2054	A(3,14) = 0.	2158	A(5,18) = 0.
2056	A(4,14) = 0.	2160	A(6,18) = 0.
2058	A(5,14) = 0.	2162	A(7,18) = 0.
2060	A(6,14) = 0.	2164	A(8,18) = 0.
2062	A(7,14) = 0.	2166	A(9,18) = 0.
2064	A(8,14) = -1.0	2168	A(10,18) = 0.
2066	A(9,14) = 0.	2170	A(11,18) = 0.
2068	A(10,14) = 0.	2172	A(12,18) = 0.
2070	A(11,14) = 0.	2174	A(13,18) = 0.
2072	A(12,14) = 0.	2176	A(14,18) = 0.
2074	A(13,14) = 0.	2178	A(15,18) = 0.
2076	GO TO 725	2180	A(16,18) = -1.0
2080	A(1,14) = 0.	2182	A(17,18) = 0.
2082	A(2,14) = 0.	2184	GO TO 725
2084	A(3,14) = 0.	725	DO 727 J=1,M
2086	A(4,14) = 0.	726	DO 727 I=1,N
2088	A(5,14) = 0.	727	PRINT 406, A(I,J)
2090	A(6,14) = 0.	728	DO 765 J=1,M
2092	A(7,14) = 0.	730	DO 765 I=1,N
2094	A(8,14) = 0.	735	IF(I-J)1230,740,740
2096	A(9,14) = 0.	740	G(I,J) = A(I,J)
2098	A(10,14) = 0.	745	DO 765 LL=1,J
2100	A(11,14) = 0.	750	IF(LL-J)755,751,751
2102	A(12,14) = -1.0	755	TERM = G(I,LL)*H(LL,J)
2104	A(13,14) = 0.	760	G(I,J) = G(I,J)-TERM
2146	GO TO 725	765	CONTINUE
1420	A(1,18) = 0.	770	DO 790 J=1,M
1421	A(2,18) = 0.	775	DO 790 I=1,N
1422	A(3,18) = -1.0	780	IF(I-J)781,785,785
1423	A(4,18) = 0.	785	PRINT 404, G(I,J)
1424	A(5,18) = 0.	790	CONTINUE
1425	A(6,18) = 0.	800	K = N+1
1426	A(7,18) = 0.	805	KK = M
1427	A(8,18) = 0.	810	DO 850 I=1,N
1428	A(9,18) = 0.	815	LK = K-I
1429	A(10,18) = 0.	820	X(LK) = H(LK,KK)
1430	A(11,18) = 0.	825	DO 850 J=1,M
1431	A(12,18) = 0.	830	LKK = KK-J
1432	A(13,18) = 0.	835	IF(LK-LKK)840,836,836
1433	A(14,18) = 0.	840	TERM = H(LK,LKK)*X(LKK)
1434	A(15,18) = 0.	845	X(LK) = X(LK)-TERM
1435	A(16,18) = 0.	850	CONTINUE
1436	A(17,18) = 0.	855	K = N+1
1437	GO TO 725	860	DO 870 I=1,N
1450	A(1,18) = 0.	865	LK = K-I
1451	A(2,18) = 0.	870	PRINT 406, X(LK)
1452	A(3,18) = 0.	874	GO TO(1550,1600,1650,900)LM
1453	A(4,18) = -1.0	900	IF(AL-H1)902,906,960
1454	A(5,18) = 0.	902	AL = AL+DLT1
1455	A(6,18) = 0.	904	GO TO 30
1456	A(7,18) = 0.	906	IF(G8-H10)908,912,960
1457	A(8,18) = 0.	908	G8 = G8+DLT10
1458	A(9,18) = 0.	910	GO TO 30
1459	A(10,18) = 0.	912	IF(B-H2)914,918,960
1460	A(11,18) = 0.	914	B = B+DLT2
1461	A(12,18) = 0.	916	GO TO 30
1462	A(13,18) = 0.	918	IF(G7-H9)920,924,960
1463	A(14,18) = 0.	920	G7 = G7+DLT9
1464	A(15,18) = 0.	922	GO TO 30
1465	A(16,18) = 0.	924	IF(G6-H8)926,930,960
1466	A(17,18) = 0.	926	G6 = G6+DLT8
1467	GO TO 725	928	GO TO 30
1480	A(1,18) = 0.	930	IF(G5-H7)932,936,960
1481	A(2,18) = 0.	932	G5 = G5+DLT7
1482	A(3,18) = 0.		

TABLE 3-1 (Continued)

```

934 GO TO 30
936 IF(G4-H6)938,942,960
938 G4 = G4+DLT6
940 GO TO 30
942 IF(G3-H5)944,948,960
944 G3 = G3+DLT5
946 GO TO 30
948 IF(G2-H4)950,954,960
950 G2 = G2+DLT4
952 GO TO 30
954 IF(G1-H3)956,960,960
956 G1 = G1+DLT3
958 GO TO 30
960 STOP
1550 IF(AL-H1)1552,1556,960
1552 AL = AL+DLT1
1554 GO TO 30
1556 IF(G2-H4)1558,1562,960
1558 G2 = G2+DLT4
1560 GO TO 30
1562 IF(B-H2)1564,1568,960
1564 B = B+DLT2
1566 GO TO 30
1568 IF(G1-H3)1570,960,960
1570 G1 = G1+DLT3
1572 GO TO 30
1600 IF(AL-H1)1602,1608,960
1602 AL = AL+DLT1
1604 GO TO 30
1608 IF(G4-H6)1610,1614,960
1610 G4 = G4+DLT6
1612 GO TO 30
1614 IF(B-H2)1616,1620,960
1616 B = B+DLT2
1618 GO TO 30
1620 IF(G3-H5)1622,1626,960
1622 G3 = G3+DLT5
1624 GO TO 30
1626 IF(G2-H4)1628,1632,960
1628 G2 = G2+DLT4
1630 GO TO 30
1632 IF(G1-H3)1634,960,960
1634 G1 = G1+DLT3
1636 GO TO 30
1650 IF(AL-H1)1652,1656,960
1652 AL = AL+DLT1
1654 GO TO 30
1656 IF(G6-H8)1658,1662,960
1658 G6 = G6+DLT8
1660 GO TO 30
1662 IF(B-H2)1664,1668,960
1664 B = B+DLT2
1666 GO TO 30
1668 IF(G5-H7)1670,1674,960
1670 G5 = G5+DLT7
1672 GO TO 30
1674 IF(G4-H6)1676,1680,960
1676 G4 = G4+DLT6
1678 GO TO 30
1680 IF(G3-H5)1682,1686,960
1682 G3 = G3+DLT5
1684 GO TO 30
1686 IF(G2-H4)1688,1692,960
1688 G2 = G2+DLT4
1690 GO TO 30
1692 IF(G1-H3)1694,960,960
1694 G1 = G1+DLT3
1696 GO TO 30
751 TERM = 0.
752 GO TO 760
1230 H(I,J) = A(I,J)/G(I,I)
1240 DO 1255 L=1,I
1245 IF(L=1)1250,1241,1241
1250 TERM = G(I,L)*H(L,J)/G(I,I)
1255 H(I,J) = H(I,J)-TERM
1260 GO TO 765
1241 TERM = 0.
1242 GO TO 1255
781 PRINT 408, H(I,J)
782 GO TO 790
836 TERM = 0.
837 GO TO 845
400 FORMAT (16,16,16,16,16,16,16)
402 FORMAT (F10.6,F10.6,F10.6,F10.6,F10.6,F10.6)
404 FORMAT (F10.6)
406 FORMAT (F10.6)
408 FORMAT (F10.6)
END

```

loading considered. All seven words are fixed point numbers as indicated in Fig. 3-3, which represents the input data for a four-span frame with a horizontal force at joint 2. The following symbols are introduced from the program

N = Number of rows in equilibrium matrix

M = Number of columns in equilibrium matrix

LN, LNN, LLN, LLNN, LM = Numbers which specify the type of frame and type of loading

(1) One-Span Frame

N = 5

M = 6

LN = 1, 2 or 3 for uniform loading, a horizontal force at Joint 1 or 2, respectively

LNN = Any number from 1 - 5

LLN = Any number from 1 - 6

LLNN = Any number from 1 - 6

LM = 1

(2) Two-Span Frame

N = 9

M = 10

LN = 4

LNN = 1, 2, 3 or 4 for uniform loading, a horizontal force at Joint 1, 2 or 3, respectively

LLN = Any number from 1 - 6

LLNN = Any number from 1 - 6

LM = 2

(3) Three-Span Frame

N = 13

M = 14

LN = 4

LNN = 5

LLN = 1, 2, 3, 4 or 5 for uniform loading, a horizontal force  
at Joint 1, 2, 3 or 4, respectively

LLNN = Any number 1 - 6

LM = 3

(4) Four-Span Frame

N = 17

M = 18

LN = 4

LNN = 5

LLN = 6

LLNN = 1, 2, 3, 4, 5 or 6 for uniform loading, a horizontal  
force at Joint 1, 2, 3, 4 or 5, respectively

LM = 4

Word 1	Word 2	Word 3	Word 4	Word 5	Word 6	Word 7	
N	M	LN	LNN	LLN	LLNN	LM	
□□□□17	□□□□18	□□□□4	□□□□5	□□□□6	□□□□3	□□□□4	

FIG. 3-3. TYPICAL INPUT DATA-FORMAT I

The other five cards contain values of parameters. Each card consists of six words which are floating point numbers (Fig. 3-4). For analyzing a specific frame, the increments may be read in as zero, and the final values may be the same as the initial values.

Initial Value	Increment	Final Value
$\left\{ \begin{array}{l} \text{AL} = \alpha \text{ (Word 1)} \\ \text{B} = \beta \text{ (Word 4)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{DLT 1 (Word 2)} \\ \text{DLT 2 (Word 5)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{H1 (Word 3)} \\ \text{H2 (Word 6)} \end{array} \right.$
$\left\{ \begin{array}{l} \text{G1} = r_1 \text{ (Word 1)} \\ \text{G2} = r_2 \text{ (Word 4)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{DLT 3 (Word 2)} \\ \text{DLT 4 (Word 5)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{H3 (Word 3)} \\ \text{H4 (Word 6)} \end{array} \right.$
$\left\{ \begin{array}{l} \text{G3} = r_3 \text{ (Word 1)} \\ \text{G4} = r_4 \text{ (Word 4)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{DLT 5 (Word 2)} \\ \text{DLT 6 (Word 5)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{H5 (Word 3)} \\ \text{H6 (Word 6)} \end{array} \right.$
$\left\{ \begin{array}{l} \text{G5} = r_5 \text{ (Word 1)} \\ \text{G6} = r_6 \text{ (Word 4)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{DLT 7 (Word 2)} \\ \text{DLT 8 (Word 5)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{H7 (Word 3)} \\ \text{H8 (Word 6)} \end{array} \right.$
$\left\{ \begin{array}{l} \text{G7} = r_7 \text{ (Word 1)} \\ \text{G8} = r_8 \text{ (Word 4)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{DLT 9 (Word 2)} \\ \text{DLT 10 (Word 5)} \end{array} \right.$	$\left\{ \begin{array}{l} \text{H9 (Word 3)} \\ \text{H10 (Word 6)} \end{array} \right.$

Word 1	Word 2	Word 3	Word 4	Word 5	Word 6	
AL	D L T 1	H1	B	DLT 2	H2	
0.10000000	0.10000000	1.00000000	0.10000000	0.10000000	0.50000000	

FIG. 3-4. TYPICAL INPUT DATA-FORMAT II

### 3-6. Output Data

All output data are in floating point form, and consist of the following:

- (1) Equilibrium matrix
- (2) G- and H-matrices
- (3) Final end moments

The equilibrium matrix and the G- and H-matrices are obtained for checking purposes only.

## CHAPTER IV

### APPLICATION

#### 4-1. Procedure

A procedure for analyzing continuous frames is outlined as follows:

##### (A) General System of Loads

- (1) Determine the values of  $\alpha$ ,  $\beta$ , and  $r$ 's.
- (2) Compute end moments at each joint due to applied loads, assuming all joints locked against translation.
- (3) Compute horizontal thrusts and shears.
- (4) Calculate the balancing force at each joint

$$-P_j = H_{ji} + H_{j0} + H_{jk}$$

- (5) Compute the end moments due to the balancing force  $P_j$  at each joint (Step 4) by using the developed computer program with the proper input data.
- (6) Obtain the final end moments by adding algebraically the results of Steps 2 and 5.

This procedure is illustrated by Example 3.

##### (B) Uniformly Distributed Load or Horizontal Force Applied at a Joint

- (1) Determine the values of  $\alpha$ ,  $\beta$ , and  $r$ 's.
- (2) Compute the end moments due to these special loads by using the developed computer program with the proper input data.

This procedure is illustrated by Examples 1 and 2.



## 4-2. Numerical Examples

Example 1 - A one-span gable frame, with dimensions and load, as shown in Fig. 4-1, is considered.

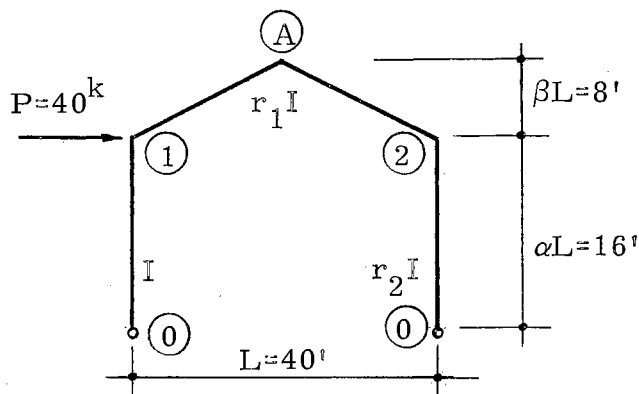


FIG. 4-1. ONE-SPAN GABLE FRAME

Parameters:

$$\alpha = 0.4 \quad \beta = 0.2 \quad r_1 = 1.2 \quad r_2 = 1.0 \quad PL = 1600$$

Input Data:

$$N = 5$$

$$M = 6$$

$$LN = 2$$

$$LNN = \text{Any fixed point number } 1 - 5$$

$$LLN = LLNN = \text{Any fixed point number } 1 - 6$$

$$LM = 1$$

$$AL = H1 = 0.4$$

$$B = H2 = 0.2$$

$$G1 = H3 = 1.2$$

$$G2 = H4 = 1.0$$

$$G3 = H5 = G4 = H6 = . . . = \text{Any floating point number}$$

$$DLT1 = DLT2 = . . . = 0.0$$

Final End Moments:

	<u>This Thesis</u>	<u>From Hale (6)</u>
$M_{10}$	$+0.235452 (1600) = +376.40^{\text{k-ft}}$	
$M_{1A}$	$+0.235452 (1600) = +376.40^{\text{k-ft}}$	$+0.235452(1600) = +376.40^{\text{k-ft}}$
$M_A$	$-0.046821 (1600) = -74.95^{\text{k-ft}}$	
$M_{2A}$	$-0.164547 (1600) = -263.50^{\text{k-ft}}$	$+0.164547(1600) = +263.50^{\text{k-ft}*}$
$M_{20}$	$-0.164547 (1600) = -263.50^{\text{k-ft}}$	

\* Different sign convention

Example 2 - A two-span gable frame with constant moment of inertia is considered. Dimensions and loads are shown in Fig. 4-2.

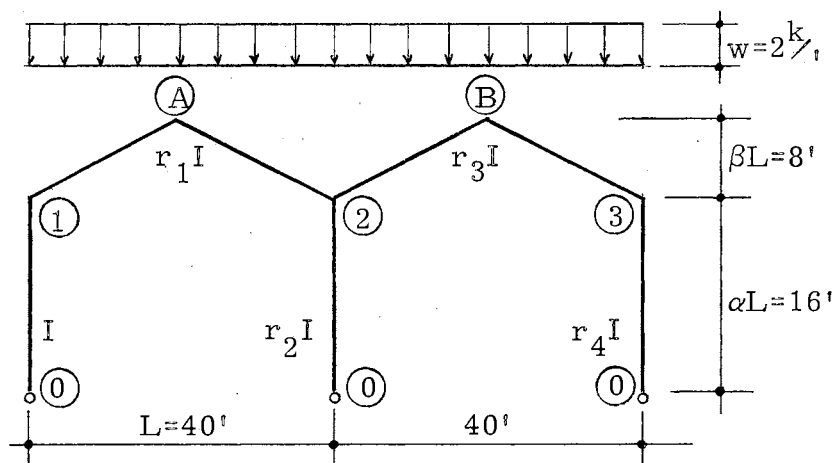


FIG. 4-2. TWO-SPAN GABLE FRAME

Parameters:

$$\alpha = 0.4 \quad \beta = 0.2 \quad r_1 = r_2 = r_3 = r_4 = 1.0 \quad wL^2 = 3200$$

Input Data:

$$N = 9$$

$$M = 10$$

$$LN = 4$$

$$LNN = 1$$

$$LLN = LLNN = \text{Any fixed point number } 1 - 6$$



$$(1) \quad \alpha = 0.4$$

$$\beta = 0.2$$

$$r_2 = r_4 = 0.8$$

$$r_1 = r_3 = r_5 = 0.6$$

$$r_6 = 1.0$$

(2) The end moments assuming all joints locked against translation are obtained by ordinary moment distribution for straight member.

The stiffness factors, distribution factors, carry-over factors, and fixed-end moments are tabulated numerically in Table 4-1.

The distribution of fixed-end moments is shown in Table 4-2.

(3) End shears and horizontal thrusts, due to end moments (Table 4-2) and drift loads, are obtained as shown in Fig. 4-4. Slope-deflection sign convention is adopted, positive signs are indicated on the figure.

$$H_{10} = + 0.85^k$$

$$H_{3B} = -25.00^k$$

$$H_{1A} = +27.40^k$$

$$H_{30} = + 2.93^k$$

$$H_{3C} = +25.00^k$$

$$H_{2A} = -28.70^k$$

$$H_{20} = + 2.84^k$$

$$H_{4C} = -25.00^k$$

$$H_{2B} = +25.00^k$$

$$H_{40} = + 3.59^k$$

(4) Balancing forces are shown in Fig. 4-4. The balancing force matrix

is

$$[P] = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} -28.25 \\ + 0.86 \\ - 2.93 \\ +21.41 \end{bmatrix} k$$

TABLE 4-1 ELASTIC CONSTANTS

Member	Stiffness Factor	Distribution Factor	Carry-Over Factor	Fixed-End Moment
0-1	0	0	0	0
1-0	0.1875	0.6270	0	0
1-A	0.1114	0.3730	0.5000	0
A-1	0.1114	0.5000	0.5000	0
A-2	0.1114	0.5000	0.5000	-66.67
2-A	0.1114	0.2990	0.5000	+66.67
2-0	0.1500	0.4020	0	0
0-2	0	0	0	0
2-B	0.1114	0.2990	0.5000	0
B-2	0.1114	0.5000	0.5000	0
B-3	0.1114	0.5000	0.5000	-66.67
3-B	0.1114	0.2990	0.5000	+66.67
3-0	0.1500	0.4020	0	0
0-3	0	0	0	0
3-C	0.1114	0.2990	0.5000	0
C-3	0.1114	0.5000	0.5000	0
C-4	0.1114	0.5000	0.5000	-66.67
4-C	0.1114	0.3730	0.5000	+66.67
4-0	0.1875	0.6270	0	0
0-4	0	0	0	0

TABLE 4-2. DISTRIBUTION OF FIXED-END MOMENTS

	10	1A	A1	A2	2A	20	2B	B2	B3	3B	30	3C	C3	C4	4C	40
-D's	-.627	-.373	-.5	-.5	-.299	-.402	-.299	-.5	-.5	-.299	-.402	-.299	-.5	-.5	-.373	-.627
C's		.5	.5	.5	.5		.5	.5	.5	.5		.5	.5	.5	.5	
FM's			+33.34	+33.34	-19.93	-26.81	-19.93	+33.34	+33.34	-19.93	-26.81	-19.93	+33.34	+33.34	-24.90	-41.77
	-10.45	+16.67 -6.22	+4.98	-9.97 +4.98	+16.67 -9.97	-13.40	+16.67 -9.97	-9.97 +9.97	-9.97 +9.97	+16.67 -9.97	-13.40	+16.67 -9.97	-9.97 +11.21	-12.45 +11.21	+16.67 -6.22	-10.45
	-1.56	+2.49 -.93	-3.11 +4.04	-4.98 +4.04	+2.49 -2.23	-3.01	+4.98 -2.23	-4.98 +4.98	-4.98 +4.98	+4.98 -3.17	-4.25	+5.61 -3.17	-4.98 +4.04	-3.11 +4.04	+5.61 -2.09	-3.52
	-1.27	+2.02 -.75	-.47 +.79	-1.11 +.79	+2.02 -1.35	-1.81	+2.49 -1.35	-1.11 +1.35	-1.59 +1.35	+2.49 -1.35	-1.81	+2.02 -1.35	-1.59 +1.32	-1.05 +1.32	+2.02 -.75	-1.27
	-.24	+.39 -.15	-.38 +.52	-.67 +.52	+.39 -.31	-.45	+.67 -.31	-.67 +.67	-.67 +.67	+.67 -.4	-.53	+.66 -.4	-.67 +.52	-.38 +.52	+.66 -.25	-.41
RM's	-13.52	+13.52	+39.71	-26.94	-12.62	-45.48	-8.98	+33.58	+33.09	-10.01	-46.80	-9.86	+33.22	+33.44	-9.25	-57.42
M's	-13.52	+13.52	+39.71	-39.73	+54.05	-45.48	-8.98	+33.58	-33.58	+56.66	-46.80	-9.86	+33.22	-33.23	+57.42	-57.42

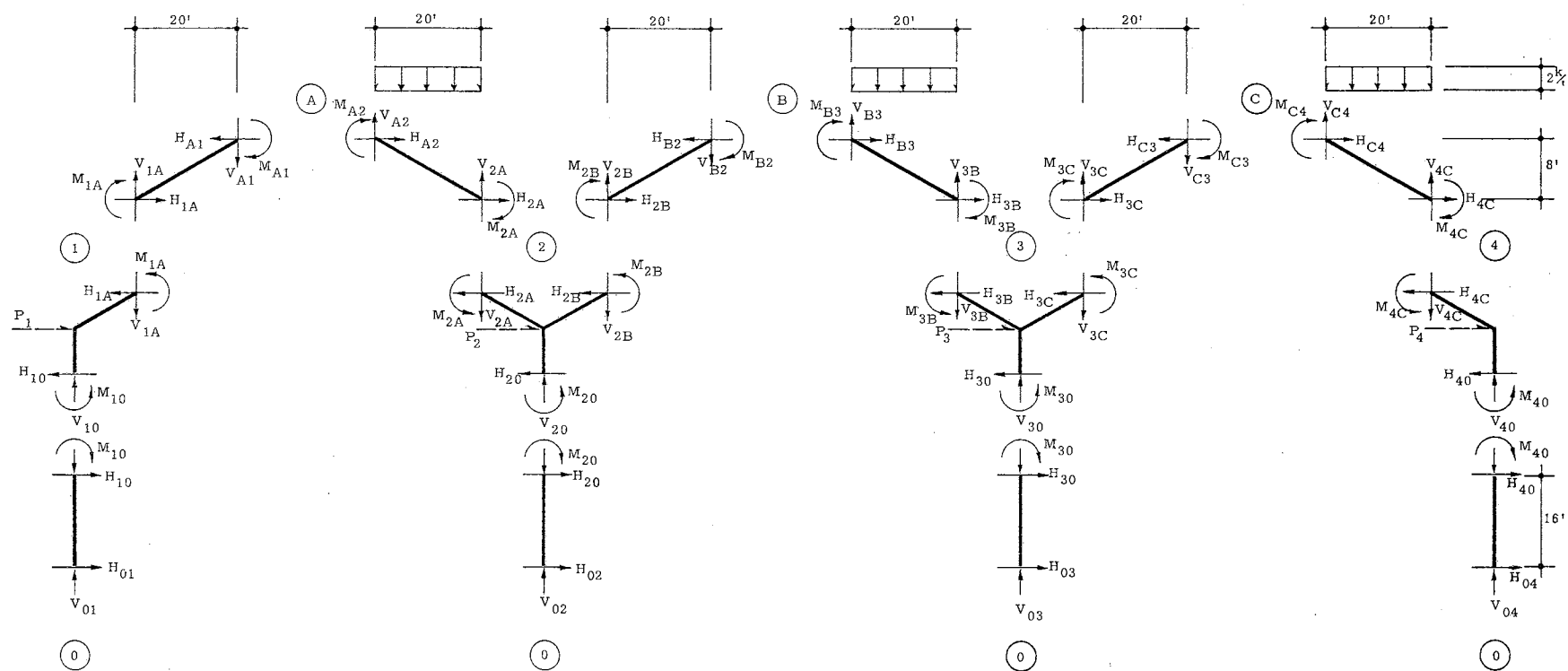


FIG. 4-4. FREE-BODY DIAGRAM

The total horizontal force matrix is

$$[P^T] = \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \\ P_4^T \end{bmatrix} = \begin{bmatrix} -28.25 + 40 \\ + 0.86 \\ - 2.93 \\ +21.41 \end{bmatrix} = \begin{bmatrix} +11.75 \\ + 0.86 \\ - 2.93 \\ +21.41 \end{bmatrix} \quad k$$

(5) End moments due to total horizontal forces at joints:

$$[M^{(P)}] = [m]^* [P^T_L]$$

	Col. 1	Col. 2	Col. 3	Col. 4		
$M_{10}^{(P)}$	+ .143687	+ .063963	- .060336	- .053696	+470.0	+108.63 k-ft
$M_{1A}^{(P)}$	+ .143687	+ .063963	- .060336	- .053696	+ 34.4	+108.63
$M_A^{(P)}$	- .060391	+ .024015	- .018079	- .016435	+117.2	- 15.64
$M_{2A}^{(P)}$	- .008158	- .079894	+ .084513	+ .074521	-857.0	- 60.56
$M_{20}^{(P)}$	- .107069	- .173562	+ .102138	+ .095547		-125.80
$M_{2B}^{(P)}$	- .098911	- .093667	+ .017624	+ .021026		- 65.65
$M_B^{(P)}$	+ .035679	+ .043216	+ .043216	+ .035679		- 7.24
$M_{3B}^{(P)}$	+ .021026	+ .017624	- .093667	- .098911		+84.25
$M_{30}^{(P)}$	+ .095547	+ .102138	- .173562	- .107069		+119.77
$M_{3C}^{(P)}$	+ .074521	+ .084513	- .079894	- .008158		+ 35.52
$M_C^{(P)}$	- .016435	- .018079	+ .024015	- .060391		+ 57.20
$M_{4C}^{(P)}$	- .053696	- .060336	+ .063963	+ .143687		-142.96
$M_{40}^{(P)}$	- .053696	- .060336	+ .063963	+ .143687		-142.96



\* $[m]$  = End moment coefficient matrix obtained from the developed computer program. Columns 1, 2, 3, and 4 are end moments due to a unit force applied at Joints 1, 2, 3, and 4, respectively.

(6) Final End Moments:

$$[M] = [M^{(0)}] + [M^{(P)}]$$

$$\begin{bmatrix} M_{10} \\ M_{1A} \\ M_A \\ M_{2A} \\ M_{20} \\ M_{2B} \\ M_B \\ M_{3B} \\ M_{30} \\ M_{3C} \\ M_C \\ M_{4C} \\ M_{40} \end{bmatrix} = \begin{bmatrix} +13.52 \\ +13.52 \\ -39.71 \\ -54.05 \\ -45.48 \\ +8.98 \\ +33.58 \\ +56.66 \\ +46.80 \\ -9.86 \\ -33.22 \\ -57.42 \\ -57.42 \end{bmatrix} + \begin{bmatrix} +108.63 \\ +108.63 \\ -15.64 \\ -60.56 \\ -125.80 \\ -65.65 \\ -7.24 \\ +84.25 \\ +119.77 \\ +35.52 \\ -57.20 \\ -142.96 \\ -142.96 \end{bmatrix} = \begin{bmatrix} +122.15 \\ +122.63 \\ -55.35 \\ -114.61 \\ -171.28 \\ -56.67 \\ +26.34 \\ +140.91 \\ +166.57 \\ +25.66 \\ -90.42 \\ -200.38 \\ -200.38 \end{bmatrix} \text{ k-ft}$$

## CHAPTER V

### SUMMARY AND CONCLUSIONS

#### 5-1. Summary

The primary objective of this study was to develop a general computer program for the analysis of one-, two-, three-, and four-span gable frames. The structures considered are hinged at the base and loaded by:

- (1) Uniformly distributed load, or
- (2) Horizontal force applied at each joint independently.

The frames are symmetric in overall geometry, but may be unsymmetric in sectional properties because of different moments of inertia assumed for each member.

The equilibrium matrix for a four-span frame, expressed in terms of parameters  $\alpha$ ,  $\beta$ , and  $r$ 's, has been developed in Chapter II. The matrices for one-, two-, and three-span frames can be obtained directly from the larger matrix by eliminating certain rows and columns. A general computer program to formulate and solve this equilibrium matrix for the various frames and types of loading has been presented in Chapter III.

A procedure of application has been illustrated by numerical examples in Chapter IV.

#### 5-2. Conclusions

The concept of using a one-span gable frame as a basic unit for analyzing continuous gable frames has been established in this thesis.

The computer program developed can be adopted and utilized as a tool for evaluating end moment coefficients for one-, two-, three-, and four-span gable frames, in the form of tables, by varying the various parameters through practical ranges.

Similar computer programs could be developed to encompass a wider range of applicability, including variation in span lengths, certain types of unsymmetry, and additional loading conditions.

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